On the Throughput Optimality of Distributed MAC Protocols for Directional Antennas

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Abstract—**Existing distributed medium access control (MAC) protocols for wireless networks with directional antennas do not achieve the maximum possible throughput. This paper theoretically proves that existing MAC protocols such as those based on tones or IEEE 802.11 may perform arbitrarily worse as compared to a throughput-optimal MAC protocol. Next, we present a distributed scheme for achieving throughput optimality in wireless networks with directional antennas. Simulation results are presented to show the performance improvements facilitated by the throughput-optimal MAC protocol over existing protocols.**

I. Introduction

Wireless networks with directional antennas have the potential to achieve a number of benefits such as longer transmission range, reduced interference, and higher spatial reuse. In order to achieve the performance gains possible with directional antennas, the transmissions from different nodes have to be coordinated carefully in order to minimize the interference and maximize the number of simultaneous transmissions. However, MAC layer protocols for use with directional antennas have to overcome a number of challenges such as deafness and hidden terminals [1], [2]. While numerous MAC protocols have been proposed in literature for directional antennas (see [11] for a survey), none of these are throughput-optimal in the sense of ensuring stability for all arrival vectors in the capacity region (see Section II for a formal definition). In this paper we first present a result on the performance of maximal schedulers and then address the problem of developing a throughput optimal MAC protocol for wireless networks with directional antennas.

The problem of developing MAC protocols or MAC layer schedulers for wireless networks with directional antennas has received considerable attention over the last decade. Broadly, such protocols or schedulers may be categorized as either random access based or centralized. Our interest in this paper is on the random access based or distributed MAC protocols and the protocols proposed in literature in this category may be broadly classified as carrier sensing multiple access (CSMA) based or tone based [3], [4], [5], [6], [7], [8], [9]. There exists a large body of work on MAC protocols for directional antennas (see [11] for a survey). However, a fundamental feature of all these protocols is that they are essentially *maximal* schedulers at best. Maximal scheduling only ensures that if a node u has a packet for node v, then either the transmission from u to v is scheduled or a node pair than causes interference to the node pair (u,v) is scheduled. In general, the performance of a protocol such as that based on CSMA with collision avoidance (CA) may be even worse than a maximal scheduler (when there are collisions).

While maximal schedulers have low complexity, they usually achieve only a small fraction of the capacity region [14], [12]. Thus the vast number of existing MAC protocols for directional antennas are not throughput-optimal, and their performance leaves much to be desired. While recently throughput-optimal CSMA type random access protocols have been proposed for omnidirectional antennas [13], [12] such protocols are absent for networks with directional antennas.

This paper addresses the problem of developing *distributed, throughput-optimal* MAC protocols for wireless networks with directional antennas. We fist show that traditional maximal scheduling based MAC protocols cannot guarantee any constant fraction of the achievable maximum throughput region, and their performance can be arbitrarily bad. Next, we present a distributed MAC layer protocol that is guaranteed to be throughput-optimal. Simulation results are presented to show the improvement in the achieved throughput with the throughput-optimal protocol over existing protocols.

The rest of the paper is organized as follows. Section II presents a survey of related literature, and the background information and the assumptions made in this paper. Section III shows that the performance of maximal schedulers for directional antennas may perform arbitrarily worse than throughput-optimal schedulers. A distributed, throughput-optimal MAC protocol for directional antennas is presented in Section IV. Simulation results to evaluate and compare the performance of the throughput-optimal MAC protocol are presented in Section V. Section VI concludes the paper.

II. Background and Related Work

Existing distributed MAC protocols for directional antennas can be broadly classified as single channel or multi-channel, tone based. Early work in this direction was based on modifying the IEEE 802.11 MAC protocol for directional antennas. Omni-directional transmission of RTS and CTS packets followed by directional transmission of the data and ACK packets is suggested in [3]. This protocol is modified in [4] where the RTS packet is sent directionally while the CTS packet is sent omni-directionally. Directional virtual carrier sensing is proposed in [5] while the directional MAC protocol (where all MAC layer operations are done in the directional mode) is proposed in [6]. Other variants of MAC protocols based on use the RTS and CTS packets are presented in [7], [8]. In [9] a solution in proposed for the deafness and hidden terminal problem using a single channel and a single radio interface.

The use of tones to signal a busy medium has also been proposed as an alternative means of reserving the channel. The tone based protocol in [10] requires multiple transceivers, capable of transmitting data and busy tones simultaneously. The tone based protocol in [2] uses omni-directional tones after the data transmission in order to allow nodes suffering from deafness to go into repeated backoffs.

All of the protocols listed above at best achieve maximal scheduling and are not throughput-optimal in the sense that they do not achieve the entire capacity region.

A. Background

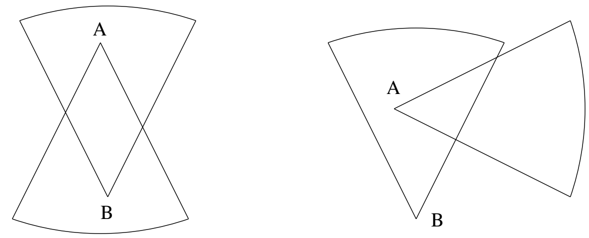
We consider a network where each node is equipped with a single, directional antenna. We assume that the nodes use an adaptive antenna array and are capable of steering its radiation pattern in any arbitrary direction. The antennas are also assumed to be capable of placing nulls in the direction of the interferences and thus we do not explicitly account for sidelobes in the radiation patterns. Each session I with a packet transmission involves two nodes: the source S and the destination D. Thus a session may be represented as a 3-tuple (I, S, D).

We model a wireless network as a graph $G = (V, E)$, where V is the set of nodes and E is the set of links. The transmission region of each node is assumed to have the shape of a circular sector, with a central angle of θ. As shown in Figure 1, two nodes A and B interfere with each other if and only if node A lies in the transmission region of node B *and* node B lies in the transmission region of node A. We term two nodes that interfere with each other as *neighbors*. By assuming bidirectional symmetric communication, if node A is node B's neighbor, then node B is also node A's neighbor. If A and B are neighbors, there is a link $(A, B) \in E$. We denote the neighborhood of node A as N_A, defined as the set of nodes that are in A's transmission range *and* cause interference with the transmission involving node A. In addition, it is assumed that each node has a single transceiver (transmitter/receiver). Thus each node can only participate in one session at a time. Then a session (I_i, S_i, D_i) is successful when none of the nodes in this session is participating in other sessions *and* if none of the neighbors of S_i and D_i transmit in this slot. The *conflict set* of session I_i is then,

$$C(I_i) = \{I_j\text{: } I_j \text{ shares a common node with } I_i, \\ (S_j \text{ or } D_j) \in (N_{S_i} \cup N_{D_i})\} \quad (1)$$

If $I_j \in C(I_i)$ and $I_i \in C(I_j)$, sessions I_i and I_j are also defined as *neighbors*.

We call a set of sessions that can be simultaneously active without interfering with each other a *feasible schedule*. A schedule is represented by a $|E|$-dimensional vector $\sigma \in \{0, 1\}^{|E|}$ whose i-th element equals 1 if link i is included in the session and equals 0 otherwise. The set of all possible



(a) Interference

(b) No interference

Fig. 1. Interference model for directional antennas.

feasible schedules is denoted by $\mathcal{S}$. The objective of the MAC protocol is to select a schedule from the set $\mathcal{S}$ for use by the nodes. Since our focus is at the MAC layer, we only consider single-hop traffic in this paper.

We define the *capacity region* of a network as the set of all arrival rates λ for which there exists a scheduling algorithm that results in stable or bounded queues at all nodes. From [15], the capacity region of a network may be defined as

$$\mathbf{\Lambda} = \{\lambda | \lambda \geq 0 \;\; and \;\; \exists \mu \in Co(\mathcal{S}), \lambda < \mu\}, \quad (2)$$

where $Co(\mathcal{S})$ is the convex hull of the set of feasible schedules in $\mathcal{S}$.

Finally, a scheduling algorithm is called *throughput-optimal* (or that it achieves the *maximum throughput*) if it ensures that the network is stable for all arrival rates in $\mathbf{\Lambda}$.

III. Performance of Maximal Scheduling based Protocols

Let the number of sessions in conflict set $C(I_i)$ that can be scheduled at the same time (but not with session I_i) be defined as the *conflict degree* of conflict set $C(I_i)$. Denote the maximum conflict degree in the network as $K(\mathcal{N})$. In this paper we characterize $K(\mathcal{N})$ in the context of wireless networks with directional antennas to show that when directional antennas are used, distributed maximal scheduling algorithms cannot achieve any constant fraction of the maximum throughput region (i.e. its performance can be arbitrarily worse than a maximum throughput scheduler).

In a wireless network $\mathcal{N}$, let λ_i be the *arrival rate* of session I_i, $i = 1, \cdots, N$. Define $\vec{\lambda}$ as the N-dimensional *arrival rate vector* whose components are the arrival rates of the sessions. A network is said to be *stable* if the arrival rate of each session equals its departure rate. The *throughput region* of a maximal scheduling policy π_{MS}, denoted as Λ^{MS}, is the set of arrival rate vectors such that the network is stable under π_{MS}. Also, an arrival rate vector $\vec{\lambda}$ is said to be *feasible* if it falls in the throughput region of some scheduling policy. The *maximum throughput region* of the network $\mathcal{N}$ is the set of all feasible rate vectors from all possible policies and is denoted by Λ.

We now state and prove the main result of this paper that quantifies $K(\mathcal{N})$ for wireless networks with directional antennas and thus also quantifies the guaranteed throughput region for maximal scheduling.

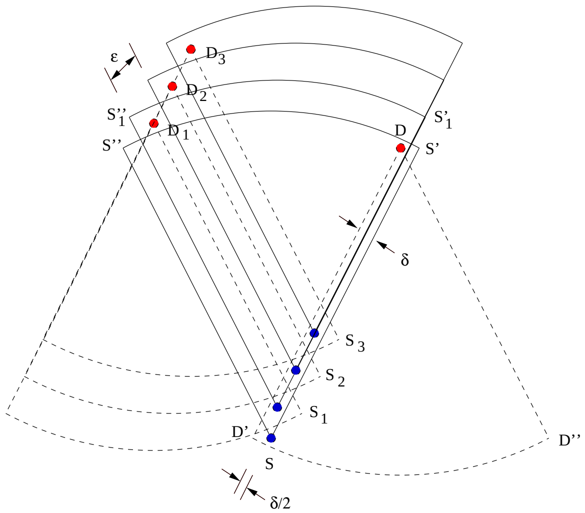


Fig. 2. Construction of a scenario with an arbitrary number of non-interfering source-destination pairs.

Theorem 1: If the same frequency and equal power are used by all nodes and bi-directional communication is involved, then, given any constant Z, there exists a wireless network $\mathcal{N}$ with directional antennas such that $K(\mathcal{N}) > Z$.

Proof: We prove the result above by construction. Consider a source (S) and destination (D) pair involved in a session as shown in Figure 2. To prove the result, we will now show that it is possible to have an arbitrary number of other sessions or source-destination pairs (with the i-th such pair denoted by S_i-D_i) that interfere with the original pair (S-D) but not with each other. Thus we will show that the conflict degree of a session may be arbitrary large.

Consider the session corresponding to the nodes S-D in Figure 2, the boundaries of whose sectors defining the transmission regions are parallel to each other, and separated by δ meters. We denote the sector corresponding to node S by the region enclosed by points S, S', S'' and the sector corresponding to node D by D, D', D'' respectively, as shown in Figure 2. Now, consider a session with nodes S_1-D_1 such that node S_1 is placed at a distance of $\delta/2$ meters between the two parallel lines SS' and DD', and a distance of ϵ meters from the arc $D'D''$. Similarly, node S_2 is located on the line S_1D at a distance of ϵ meters from S_1. Proceeding along the same lines, node S_i is placed at a distance of $(i-1)\epsilon$ meters from node S_1.

Node D_1 is placed at a distance of $\epsilon/2$ meters from the line S_1S_1'' and $\delta/2$ meters from the arc $S_1'S_1''$. Nodes D_2, D_3 and so on are also placed following the same guidelines such that the nodes $D_1, D_2, \cdots$ fall on a straight line and the distance between successive nodes is ϵ meters. Note that the node D interferes with each of the nodes $S_1, S_2, \cdots$ and thus the nodes $S_1, S_2, \cdots$ belong to the conflict set of the session corresponding to the nodes S and D. However, as can be seen from Figure 2 the sessions corresponding to nodes S_j-D_j and S_i-D_i, for $i \neq j$, do not interfere with each other. Thus these sessions may be scheduled together. Let the transmission range of each node be R, i.e., the line segment SS' is of length R meters. Then by choosing ϵ such that $\epsilon < R/(Z+2)$, we can have more than Z sessions in the conflict set of the session corresponding to nodes S-D that may be scheduled simultaneously, i.e., the conflict degree of the session corresponding to nodes S-D is greater than Z.

Note that by choosing ϵ to be small enough, we can make the conflict degree of the network arbitrarily large. Thus the fraction of the capacity region guaranteed to be achieved by maximal scheduling can be shown to be arbitrarily small. ■

The result above implies that when directional antennas are used, it is possible to have a network with an arbitrarily large conflict degree. It is well known that in any wireless network $\mathcal{N}$ with single frequency, bi-directional, equal-power, two-terminal communication network model, if $\vec{\lambda} \in \Lambda$ in $\mathcal{N}$, then $\vec{\lambda}/K(\mathcal{N}) \in \Lambda^{MS}$ in $\mathcal{N}$ [14]. In other words, for any wireless network $\mathcal{N}$, at least $1/K(\mathcal{N})$ of the maximum throughput is guaranteed given any maximal scheduling policy. For an arbitrary session I_i in $\mathcal{N}$, there are at most $K(\mathcal{N})$ sessions in $I_i \cup C(I_i)$ that can be scheduled simultaneously. Thus, the sum of departure rates and the sum of the feasible arrival rates for $I_i \cup C(I_i)$ is at most $K(\mathcal{N})$. For an arrival rate vector $\vec{\lambda}/K(\mathcal{N})$, the sum of arrival rates for $I_i \cup C(I_i)$ is at most 1. With maximal scheduling, one session is always scheduled among $I_i \cup C(I_i)$. Thus, with $\vec{\lambda}/K(\mathcal{N})$, the departure rates are greater than or equal to the arrival rates and the network is stable.

Theorem 1 above implies that when directional antennas are used, a network $\mathcal{N}$ may have an arbitrarily large conflict degree $K(\mathcal{N})$. Thus, maximal scheduling cannot guarantee that it will achieve any given fraction of the maximum throughput region, no matter how small that fraction may be. Thus its performance may be arbitrarily worse than that of a throughput-optimal scheduler or MAC protocol. This motivates the need for the development of a throughput-optimal, distributed MAC protocol for use with directional antennas and the next section addresses this problem.

IV. A Distributed Throughput Optimal MAC Protocol

In this section we describe a throughput-optimal scheduler for use in wireless networks with directional antennas. We consider a discrete, slotted-time operation where all packets have the same size and each slot is long enough so that a node may transmit at most one packet in a slot. The proposed protocol is an extension of the protocol presented in [16].

The proposed protocol operates as follows. At the start of each slot, each node follows the protocol shown in Algorithm 1 to independently and distributedly determine if it transmits a packet in this slot. We denote the scheduling decision of node i at time t by $\sigma_i(t) \in \{0, 1\}$ with $\sigma_i(t) = 1$ implying that node i is scheduled for a transmission in time slot t and $\sigma_i(t) = 0$ implying that node i does not transmit in this slot. Also, the outcome of node i's transmission in time slot t, if any, is denoted by $O_i(t) \in \{0, 1\}$ with $O_i(t) = 1$ implying

Algorithm 1 Throughput Optimal MAC Protocol

$Q_i(t)$: `queue length at node` i `at time` t
$\sigma_i(t)$: `node` i`'s schedule at time` t
$O_i(t)$: `outcome of node` i`'s transmission (if any) at time` t

for $(t = 1; ;t{+}{+})$
  $w_i(t) = \log(Q_i(t)$
  if $\sigma_i(t-1) = 1$ **then**
    if $O_i(t-1) = 1$ **then**
      $\sigma_i(t) = \begin{cases} 1 & \text{w.p. } \frac{w_i(t)}{w_i(t)+1} \\ 0 & \text{otherwise} \end{cases}$
    else
      $\sigma_i(t) = 0$
    end if
  else
    if $\cup_{j \in \mathcal{N}(i)} \sigma_j(t-1) = 0$ **then**
      $\sigma_i(t) = \begin{cases} 1 & \text{w.p. } \frac{1}{K(\mathcal{N})} \\ 0 & \text{otherwise} \end{cases}$
    else
      $\sigma_i(t) = 0$
    end if
  end if
end for

that the transmission was successful and $O_i(t) = 0$ implying that the transmission was unsuccessful (due to a collision). In addition, the scheduling decision at node i is also based on its queue length, $Q_i(t)$. Finally, the set of nodes which are within the transmission range of node i is denoted by $\mathcal{N}(i)$.

At the start of each slot, node i calculates its weight as $w_i(t) = \log Q_i(t)$. Then, if node i had transmitted in the previous slot (i.e. $\sigma_i(t-1) = 1$), it first checks if the last transmission was successful (i.e. $O_i(t-1) = 1$). In case the transmission in the previous slot was successful, node i schedules a transmission in the current slot with probability (w.p.) $w_i(t)/(w_i(t)+1)$. In case the transmission in the last slot was unsuccessful, node i defers from transmitting in this slot. Finally, if node i did not transmit in the previous slot, its decision to transmit or not in the current slot is based on the activity of its neighbors in the previous slot. If none of the neighbors of node i transmitted in the previous slot, then node i transmits in the current slot with probability $1/K(\mathcal{N})$. On the other hand, if any of its neighbors transmitted in the previous slot, node i refrains from transmitting in the current slot. The transmission probability of $1/K(\mathcal{N})$ is chosen so that the time required for a node to successfully access the channel is minimized. Note that if a node is not aware of the exact number of the neighbors that are in its conflict set, it can set the transmission probability to $1/2$ instead of $1/K(\mathcal{N})$. This only affects the convergence time and not the optimality of the algorithm.

The throughput-optimality of the MAC protocol described above can be shown by following the techniques presented in [16]. We present a brief outline to the proof technique and the detailed proof is omitted due to space limitations. The optimality of the MAC protocol described in Algorithm 1 can be established by showing that the weighted sum of the schedules generated by the protocol, with the weights given by $\log\log$ of the queue lengths at the nodes, is *close* to that obtained by the schedule (from the set of all feasible schedules $\mathcal{S}$) with the maximum weighted sum, i.e., by showing show that, $\sum_i \sigma_i(t) \log\log Q_i(t)$ is close to $\max_{s \in \mathcal{S}} \sum_i s_i \log\log Q_i(t)$, on average, for all large enough t. In other words, the MAC protocol picks schedules which are close to the maximum weight feasible schedule when the node weights are $\log\log$ of the queue lengths at the nodes. Now, when the schedules chosen follow such a property, it is well know that $\sum_i \int_0^{Q_i(t)} \log\log y dy$ is a Lyapunov function of the queue length and the function decreases by at least a fixed amount when the arrival vector lies inside the capacity region, i.e., $\lambda \in \mathbf{\Lambda}$ [17]. This in turn can be used to show that the network can be characterized by a positive recurrent Markov chain, which in turn shows the optimality of the MAC protocol.

V. Simulation Results

In this section, we evaluate the performance of the throughput-optimal MAC protocol and compare it against existing protocols. The simulations were conducted using code written in C. We consider a 1000×1000 meter region where nodes are deployed randomly. The transmission range of each node is assumed to be 250 meters unless otherwise specified. Each node picks another node in its transmission region at random to form a session. The packet size is assumed to be 512 bytes and a data rate of 11 Mbps is used. We do not consider node mobility in our simulations. Since our interest is in the MAC layer performance, we do not consider PHY layer issues such as channel noise and errors, fading etc. (these factors affect all MAC layer protocols equally). The reported throughput values are the averaged result from 20 runs with different seeds. The length of each simulation run was 100 hours.

We also compare the performance of the throughput-optimal MAC scheduler with a maximal scheduler. Instead of showing the results for the MAC protocols that have been proposed in literature, we have shown the results from a general maximal scheduler since the performance of random access based and tone based MAC protocols is bounded by the performance of maximal schedulers. A centralized maximal scheduler was used and thus there were no collisions. Thus the performance of this maximal scheduler is better than the ones possible with CSMA based MAC protocols. On the other hand, the throughput-optimal MAC protocol is distributed and has collisions. However, as we will report in the following paragraphs, the distributed, throughput-optimal MAC protocol with collisions outperforms the centralized, maximal scheduler without collisions. We used a greedy maximal scheduler where the transmission schedule in each slot was selected as follows: we first start with an empty transmission schedule. Then from the list of all sessions, a session is selected at random and

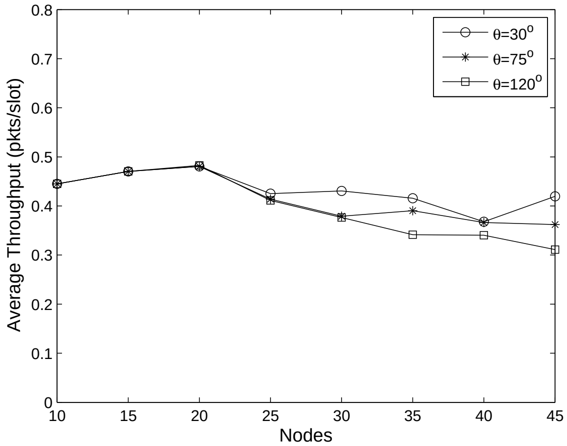


Fig. 3. Average per node throughput as a function of the number of nodes.

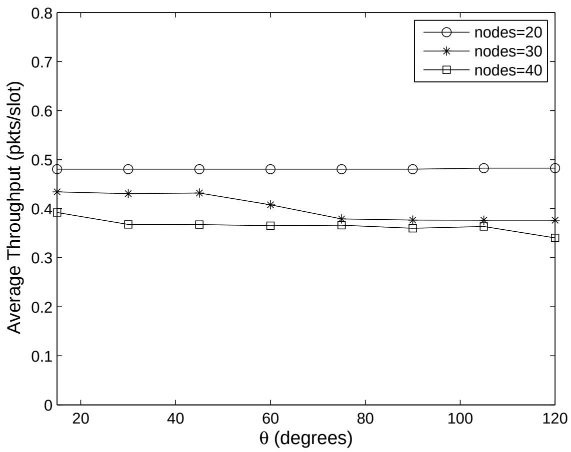


Fig. 4. Average per node throughput as a function of the beam angle.

added to the transmission schedule. Now, we select one of the remaining sessions at random and add it to the transmission schedule if it does not conflict with any of the existing sessions in the transmission schedule. If the session has a conflict, it is not added to the transmission schedule and is not considered in future iterations. The process is repeated until all sessions have been considered.

The performance of the throughput-optimal MAC protocol for various network parameters is shown in Figures 3, 4 and 5. The results for the maximal scheduler are not shown in these figures to avoid clutter. Instead, the percentage improvement in the throughput with the throughput-optimal MAC protocol over the maximal scheduler for various network parameters are shown in Tables I and II.

Figure 3 shows the average per node throughput with the throughput-optimal MAC protocol as the number of nodes in the network increases. As expected, the throughput decreases as the number of nodes increases. However, there is a slight increase in the throughput initially when the number of nodes is increased. This is because when the number of nodes is

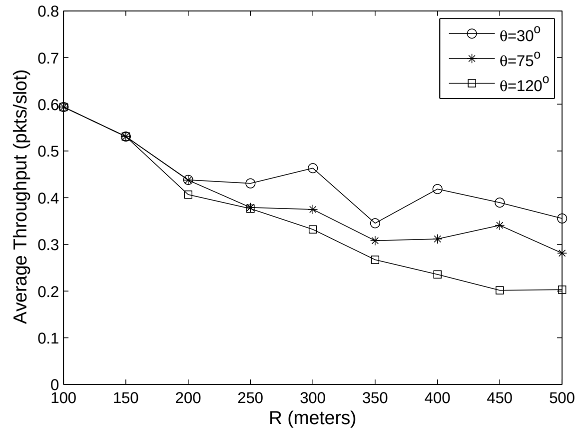


Fig. 5. Average per node throughput as a function of the transmission range.

Nodes	Percentage Improvement		
	$\theta = 30^o$	$\theta = 75^o$	$\theta = 120^o$
20	5.23%	5.23%	7.31%
30	3.68%	0.11%	5.13%
40	7.31%	4.69%	6.30%

TABLE I
PERCENTAGE IMPROVEMENT IN THE THROUGHPUT WITH THE THROUGHPUT-OPTIMAL MAC PROTOCOL OVER THE MAXIMAL SCHEDULER FOR VARIOUS NETWORK SIZES AND BEAM WIDTHS.

small, not all nodes have other nodes in their transmission and thus do not participate in any flows. As the number of nodes is increased, the number of flows increases, increasing the per node throughput. However, this also increases the interference in the network, which ultimately decreases the throughput for networks with a large number of nodes.

Figure 4 shows the throughput-optimal MAC protocol's average per node throughput for various beam angles (θ) for a network with 30 nodes. As expected, the throughput decreases as the beam angle increases since the number of nodes in the conflict set of a given node increases with the beam angle. However, as can be seen from Figure 3 the throughput is fairly independent of the beam angle when the number of nodes is small. This is because in sparse networks, increasing the beam width does not appreciably change the conflict set of a node.

Figure 5 shows the performance of the throughput-optimal MAC protocol as the transmission range of each node is increased, for a network with 30 nodes. As expected, the per node throughput decreases as the transmission range is increased and the rate of decrease is larger for large beam angles.

Tables I and II compare the performance of the throughput-optimal MAC with the centralized maximal scheduler described earlier in this section. A key observation here is that the maximal protocol used for the comparison is centralized and without any collisions. On the other hand, the throughput-optimal MAC protocol is decentralized and nodes may experience collisions with this protocol. However, it still

R	Percentage Improvement		
	$\theta = 30^o$	$\theta = 75^o$	$\theta = 120^o$
100m	57.26%	58.76%	57.39%
200m	1.36%	4.44%	3.54%
300m	4.07%	4.11%	4.96%

TABLE II
PERCENTAGE IMPROVEMENT IN THE THROUGHPUT WITH THE THROUGHPUT-OPTIMAL MAC PROTOCOL OVER THE MAXIMAL SCHEDULER FOR VARIOUS TRANSMISSION RANGES AND BEAM WIDTHS FOR A NETWORK WITH 30 NODES.

outperforms the maximal scheduler. If maximal schedulers such as directional MAC protocols based on IEEE 802.11 were used for the comparison, the performance gains would be much higher. The centralized maximal scheduler was used here for comparison to show that the throughput-optimal MAC protocol offers non-trivial gains in the performance even when compared to the best of maximal schedulers.

VI. CONCLUSION

This paper addresses the problem of throughput-optimality for MAC protocols for networks with directional antennas. We first consider the throughput of maximal schedulers such as those based on IEEE 802.11 or tones and show that theirs performance may be arbitrarily worse than that possible with a throughput-optimal MAC protocol. Next, we described a distributed protocol that achieves throughput-optimality in wireless networks with directional antennas. Simulation results are presented to show the performance improvements facilitated by the proposed protocol over existing protocols.

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