# Evaluation of the Exact Overflow Probabilites in a Space Based Multicast Switch Copy Network


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*Abstract*— **Space based multicast switches use copy networks to generate the copies of the input packets. In any time slot, the sum of the number of copies requested by the active inputs of the copy network may exceed the number of outputs and some copy requests will need to be dropped or buffered. We present an exact model to calculate the overflow probabilities in an unbuffered Lee's copy network [2]. Our exact show that the Chernoff bounds on the overflow probability is very loose and the difference can be as large as a factor of more than 10.**


## I. Introduction

Various space division packet switches architectures supporting multicasting have been proposed in literature [1], [2], [5]. The general structure of a space-based space division multicast switch is that of a copy network followed by a routing stage. In a copy network, in any slot, the sum of the number of copies requested by the active inputs may exceed its capacity and some of the copy requests may need to be queued or dropped. Our interest in this paper is on the modelling and analysis of this overflow probability in space based copy networks.

In [2], Lee studies the performance his copy network by using Chernoff bounds to calculate the overflow probabilities at each input port of the copy network. Comparision with simulation results showed that these bounds are very loose. An approximate analysis for calculating the loss probability of packet copies in the shuffle exchange copy network has been given by Liew in [3]. In this paper, we present an exact solution to calculate the overflow probabilities in Lee's copy network. This analysis may be used for the shuffle exchange based copy network with deadlock resolution. Our analysis is for the case when no buffers are present at the inputs of the copy network. For calculating the overflow probabilities, we use a technique similar to finding the normalisation constant in a product form queueing network with an inequality constraint on the state space. We then propose a contour integral approach for evaluating these probabilities.

## II. Copy Network Architecture

Lee's copy network [2] is shown in Fig. 1. Time is slotted and all the inputs are synchronised such that packet arrivals occur at the beginning of a slot. An $M \times N$ copy network, inputs numbered from 1 to $M$, works as follows: at the beginning of a slot let $c_i$ be the number of copies requested by port $i$. A running adder network determines the order in which the copy requests at the input ports are served in each slot. In the simplest case, an acyclic service discipline can be used in which the running adder begins at the top of the network and obtains the running sum of $c_i$ starting from port 1 in every slot. In this service policy port $i$ is serviced (all the copies requested by the input packet are made) if $\sum_{j=1}^{i} c_j \leq N$ in that slot. Alternatively, a cyclic service scheme that works as follows could be used. If port $i$ is the last port served in slot $n$, then in slot $n+1$ ports $i+1, i+2, ....i+k$ will be served. (All additions are modulo $M+1$ and $1 \leq k \leq M$.) In this policy, port $m$ is served if $\sum_{j=i+1}^{m} c_j \leq N$. Another variation would be to introduce fanout splitting in which a part of a copy request will be served whenever possible and the rest of the request will be served in subsequent slots.

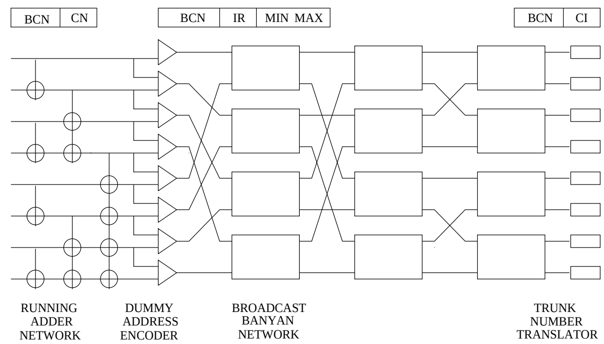


Fig. 1. Lee's Copy Network for a Multicast Packet Switch [10]

## III. Evaluation of the Overflow Probabilities

In an $M \times N$ copy network, the number of copies that can be generated in a slot is limited to $N$. In each slot the running adder starts summing the copy requests of the packets at the head of the input queues sequentially, beginning with port number 1 (acyclic service without fanout splitting). Overflow occurs in the copy network when the sum of the copy requests of the packets at the head of the queues at the input is greater than $N$. In the following, we present an exact solution for the overflow probabilities in this model.

Packet arrivals to port $i$ is a Bernoulli process with rate $\rho_i$ and the copy requests have a probability mass function $q_i(k)$. Let $X_i$ be the random variable for the number of copies requested by the input port $i$ (regardless of it being active). Then,

$$f_i(x_i) \equiv \Pr\{X_i = x_i\} = \begin{cases} 1-\rho_i, & x_i = 0 \\ \rho_i q_i(x_i), & x_i = 1, \cdots N \end{cases} \quad (1)$$

The copy request of port $i$ is served if $X_1 + X_2 + \cdots + X_i \leq N$. Thus, $P_{loss}(i)$, probability of loss at port $i$, is

$$P_{loss}(i) = 1 - \sum_{\sum_{j=1}^{i} x_j \leq N} \prod_{j=1}^{i} f_j(x_j) \quad (2)$$

The summation on the RHS of the above equation is carried out over all possible combinations of copy requests from ports 1 to $i$ that sum to less than or equal to $N$. This summation is similar to obtaining the normalisation constant in a product form queueing network with an inequality constraint on the state space. Therefore, following [4], we can obtain the $P_{loss}(i)$s as follows. Define

$$\Phi_N(k) \equiv \begin{cases} 1 & \text{for } k \leq N \\ 0 & \text{for } k > N \end{cases} \quad (3)$$

$\Phi_N(k)$ can be represented by the following contour integral in the complex plane with the unit circle as the contour of integration.

$$\Phi_N(k) = \oint \left[\frac{z^{(N+1)} - 1}{z-1}\right] \left[\frac{z^k}{z^{(N+1)}}\right] dz \quad (4)$$

We can use $\Phi_N(k)$ to represent the summation in Eqn. 2. This representation and the attendant simplifications are derived below.

$$\begin{aligned} 1 - P_{loss}(i) &= \sum_{x_1=0}^{N} \cdots \sum_{x_i=0}^{N} \prod_{k=1}^{i} f_k(x_k) \Phi_N(x_1 + \cdots + x_i) \\ &= \sum_{x_1=0}^{N} \cdots \sum_{x_i=0}^{N} \prod_{k=1}^{i} f_k(x_k) \\ &\quad \oint z^{(x_1+\cdots+x_i)} \left[\frac{z^{(N+1)} - 1}{z-1}\right] \left[\frac{1}{z^{(N+1)}}\right] dz \\ &= \oint \sum_{x_1=0}^{N} f_1(x_1) z^{x_1} \cdots \sum_{x_i=0}^{N} f_i(x_i) z^{x_i} \\ &\quad \left[\frac{z^{(N+1)} - 1}{z-1}\right] \left[\frac{1}{z^{(N+1)}}\right] dz \\ &= \oint \left[\frac{z^{(N+1)} - 1}{z-1}\right] \left[\frac{1}{z^{(N+1)}}\right] \prod_{k=1}^{i} \mathcal{F}_i(z) dz \quad (5) \end{aligned}$$

where $\mathcal{F}_i(z)$ is the moment generating function of $f_i(x_i)$. From the residue theorem, the contour integral of Eqn. 5 can evaluated by summing the residues of the integrand at poles inside $C$. It is easily seend that for the integrand in

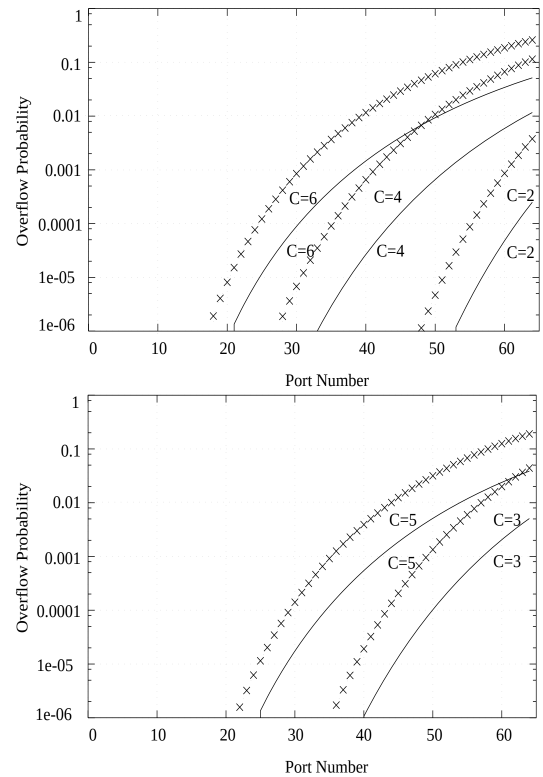


Fig. 2. Overflow probabilites in a 64 × 64 copy network with deterministic copy requests of size 2,3,4,5 and 6. The broken lines denote the Chernoff bounds while the smooth lines represent the exact results.

Eqn. 5 the only poles inside the unit circle is at $z = 0$. The Chernoff bounds on the overflow probabilities are

$$P_{Ch}(i) = \Pr\{X_1 + \cdots + X_i > N\} \leq e^{-sN} \prod_{k=1}^{i} \mathcal{F}_i(e^s) \quad (6)$$

Figure 2 compares the exact $P_{loss}$ with $P_{Ch}$.

## IV. Conclusions

In this paper, we presented an analytical tool for calculating the exact overflow probabilities in a space based multicast ATM switch. These results are a considerable improvement over the Chernoff bounds given by Lee [2]. The technique developed here can be used to carry out a queueing analysis of space based multicast switches.

## References


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