On the Convergence of MMPP and Fractional ARIMA Processes with Long-Range Dependence to Fractional Brownian Motion *

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Abstract

Though the various models proposed in the literature for capturing the long-range dependent nature of network traffic are all either exactly or asymptotically second order self-similar, their effect on network performance can be very different. We are thus motivated to characterize the limiting distributions of these models so that they lead to parsimonious modeling and a better understanding of network traffic. In this paper we consider long-range dependent arrival processes based on Markovian arrival and fractional ARIMA processes and show that the suitably scaled distributions of these processes converge to fractional Brownian motion in the sense of finite dimensional distributions. Subsequently, we prove that they also converge weakly to fractional Brownian motion in the space of continuous functions. Thus, the behavior of network elements fed with traffic from these models has similar characteristics to those fed with fractional Brownian motion under suitable limiting conditions. Specifically, tails of queues fed with these arrivals have a Weibullian shape in sharp contrast with the exponential tails of conventional queues. Also, the weak convergence results allow us to accurately estimate the loss probabilities using the expressions for storage models for fractional Browninan motion.

I Introduction

The seminal paper of [9] introduced the notion of self-similarity and long-range dependence in network traffic. This has spurred research in the area of traffic models which account for these second order statistical characteristics of network data. Wavelet models [1], Markovian arrival processes [2], the $M/G/\infty$ model [5], chaotic maps [6], Fractional Brownian motion [9], fractional ARIMA processes [9] and superposition of ON/OFF sources [14] are some of the models that have been suggested. Though all these models model the long-range dependence and show either exact or asymptotically second order self-similarity, the performance of network elements fed with these different sources differ widely. Studies in [10] and [11] show, for example, that queues fed with self-similar traffic from $M/G/\infty$ sources have an exponentially distributed queue tail while the tails of queues fed with input sources characterized by fractional Brownian motion and superposition of ON/OFF sources have a Weibullian shape. Thus additional insight into the characterization of the arriving work process is necessary to predict the queue behavior. Motivated by this striking variation in queue performance, we try to model the limiting distribution of some of these self-similar sources. Convergence of the limiting distribution to any given model for which the network element performance has already been characterized then allows us to group the model with a class of other sources, all of which lead to similar network performance.

The limiting distribution of a self-similar traffic model composed of the superposition of a large number of ON/OFF sources, whose ON/OFF periods are taken from a heavy tailed distribution is considered in [14] and [15]. The authors show that as the number of the ON/OFF sources increases and under proper scaling, the superposed process converges weakly to fractional Brownian motion in the space of continuous functions and thus it is not surprising that the tails queues fed with both these sources have similar, Weibullian shapes. In [12], the authors consider the $M/G/\infty$ source and show that under proper scaling, the process converges to a totally skewed stable Lévy motion in Skorohod space. In this paper, we first consider the Markovian arrival process [2] and show that its limiting distribution, as the number of Markovian processes in the superposition tends to infinity, converges to fractional Brownian motion in the sense of finite dimensional distributions. We also prove a similar result for fractional ARIMA [9] process based long-range dependent sources and show that their limiting distribution also

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converges to fractional Brownian motion in the sense of finite dimensional distributions. We then show that both these arrival processes also converge weakly in the space of continuous functions to fractional Brownian motion.

The rest of the paper is organized as follows. In Section II we first give a brief overview of the Markovian arrival process based long-range dependent sources introduced in [2] and prove the convergence of its finite dimensional distributions to fractional Brownian motion in Section II. In Section III, we present results for the fractional ARIMA based sources and Subsection C gives the convergence results for the finite dimensional distributions. Section IV deals with the weak convergence of both the models and Section V some numerical results for queues fed with these processes. Finally, in Section VI we present the concluding remarks and present a brief discussion on the implication of the results.

II MMPP Based Self-Similar Sources

The presence of long-range dependence in network traffic has led to the development of various traffic models which are better suited to model the second order self-similar behavior of network traffic. Most of these models are focused on capturing the first and second order characteristics of the packet count process, which, in general, is insufficient to predict queueing behavior. Also, tools for analyzing the queuing behavior of such models are still under development. As a result, the Markov Modulated Poisson Process (MMPP) based self-similar traffic model proposed in [2] becomes attractive with its already existing tools for performance measurement and the possibility in this model to capture other statistical properties in addition to the first and second order properties of the count process. To make this paper self contained, we first give a brief description of the MMPP based long-range dependent model proposed in [2].

A Preliminaries

The Markovian model of [2] is based on the superposition of a number of two-state MMPPs. The superposed process itself can be represented as a MMPP by taking the Kronecker sum of the constituent MMPPs and is thus a Markovian Arrival Process (MAP). The model fits the first and second order count process to M two-state Interrupted Poisson Processes (IPPs) and a Poisson process where the i^{th} IPP has a generator matrix Q_i and a rate matrix R_i represented as

$$\mathbf{Q_i} = \begin{bmatrix} -\mu_i^1 & \mu_i^1 \\ \mu_i^2 & -\mu_i^2 \end{bmatrix} \qquad \mathbf{R_i} = \begin{bmatrix} \lambda_i & 0 \\ 0 & 0 \end{bmatrix} \tag{1}$$

and the Poisson process has a rate $\lambda_P \geq 0$. The MAP model of the superposition of M such process can then be expressed as

$$\mathbf{Q} = \bigoplus_{i=1}^{M} Q_i \qquad \mathbf{R} = \bigoplus_{i=1}^{M} R_i \tag{2}$$

Note that the Poisson process may also be represented as the special case of an MMPP and added to the Kronecker sum of Equation 2 to obtain the complete MAP model of the arrival process. For the i^{th} IPP, the covariance function of the count process in two time slots of size Δt with $k-1$, $k>0$ time slots between them is given by

$$\begin{aligned} \gamma_i(k) &= \left[\frac{\mu_i^1\mu_i^2(\lambda_i)^2 e^{-((\mu_i^1+\mu_i^2)(k-1)\Delta t)}}{(\mu_i^1+\mu_i^2)^4}\right] \times \\ & \qquad (1-2e^{-((\mu_i^1+\mu_i^2)\Delta t)}e^{-((\mu_i^1+\mu_i^2)2\Delta t)}) \end{aligned} \tag{3}$$

which, for $(\mu_i^1+\mu_i^2)\Delta t \ll 1$, can be approximated as

$$\gamma_i(k) \approx \frac{\mu_i^1\mu_i^2(\lambda_i\Delta t)^2 e^{-((\mu_i^1+\mu_i^2)(k-1)\Delta t)}}{(\mu_i^1+\mu_i^2)^2} \tag{4}$$

In Equation (4) and subsequently in this paper, the approximation relation $f(x) \approx g(x)$ means $\lim_{x\to\infty} f(x)/g(x) = a$ for some $a \neq 0$. Also, in this paper, we use the similarity relation $f(x) \sim g(x)$ to designate $\lim_{x\to\infty} f(x)/g(x) = 1$. Since these IPPs are independent, the covariance of the superposed process can be expressed as

$$\gamma(k) = \sum_{i=1}^{M} \gamma_i(k) \tag{5}$$

B The Parameter Fitting Process

The second-order self-similarity in network packet traffic can be described by the covariance function of the count process of the packet arrivals in an interval [9]. The covariance function of such processes behave asymptotically as $cov(k) = \psi_{cov} k^{-\beta}$ where ψ_{cov} is an absolute measure of the variance and $\beta = 2-2H$, $0 \leq \beta \leq 1$ with H denoting the Hurst parameter. The model in [2] fits the covariance structure of the superposed process of a finite number (M) of IPPs to the form $\psi_{cov} k^{-\beta}$ over a given number of time scales, n. The time scales of interest in real life are limited to relatively smaller values but the fitting process can be applied to arbitrarily large time scales and values of M. The variability over a number of time scales is achieved by the choice of the time constants for each IPP through the parameters μ_i^1 and μ_i^2 which are chosen logarithmically. The fitting can be summarized as follows:

- **Step 1.** The initial choice of the modulating parameters μ_i^1 and μ_i^2 of the IPPs are chosen to satisfy $\mu_i^1 = \mu_i^2 = a^{1-i}\mu_1^1$ for $1 \leq i \leq M$ where is the logarithmic spacing parameter determined by

$$a = 10^{n/(M-1)} \qquad M > 1 \tag{6}$$

with an initial choice of $\mu_1^1 = \mu_1^2$ in the range $[0.25, 0.75]$. The initial choice of the parameters can be changed in step 3 while maintaining the sum $\mu_i^1 + \mu_i^2$, $1 \le i \le M$, constant.

- **Step 2.** The rate vector $\vec{\lambda}$ is determined from $\vec{\lambda} = k * \vec{\phi}$ where k is a normalizing constant and $\vec{\phi}$ is the vector of the relative magnitudes of the arrival rates of each IPP. For $\Delta t = 1$ and $(\mu_i^1 + \mu_i^2) \ll 1$, $1 \le i \le M$ with $\mu_i^1 = \mu_i^2$, Equation (4) can be written as

$$\gamma_i(k) \approx \frac{k^2}{4}(\phi_i)^2 e^{-((\mu_i^1+\mu_i^2)(k-1)\Delta t)} \tag{7}$$

The covariance function is fitted at M different points defined by $(\mu_i^1 + \mu_i^2)k = 1$. The equation for the i^{th} IPP is given by

$$\psi_{cov} a^{-(i-1)\beta} = \frac{k^2}{4}\sum_{j=1}^{d}(\phi_j)^2 e^{-a^{i-j}} \tag{8}$$

and these equations may be used to find $\vec{\phi}$.

- **Step 3.** This step determines the constant k and the Poisson rate λ_P and the details are given in [2]. Details regarding special cases where the parameters μ_1^1 and μ_1^2 chosen in Step 1 need to be changed can also be obtained from [2].

C Convergence to Fractional Brownian Motion

For the i^{th} IPP, let $W^{(i)}(t)$ denote the stationary binary sequence that it generates where $W^{(i)}(t) = 1$ means that there is a packet at time t and $W^{(i)}(t) = 0$ means that there is no packet. The average arrival rate for this IPP, at any given instant, can be represented as $\lambda_i\mu_i^2/(\mu_i^1+\mu_i^2)$. When there are M heterogeneous IPPs, the superposed packet count at time t is given by $\sum_{i=1}^{M} W^{(i)}(t)$ and the aggregated cumulative packet count in the interval $[0, Tt]$, $W_M(Tt)$, is then given by

$$W_M(Tt) = \int_0^{Tt}\left(\sum_{i=1}^{M} W^{(i)}(x)\right)dx \tag{9}$$

Also, let $Y^{(i)}(j)$ denote the increment process for the i^{th} IPP representing the number of arrivals in the j^{th} time unit. We now propose the following lemma.

Lemma 1: As $M \to \infty$, the increment process of $\{W_M(Tt), t \ge 0\}$ converges in the sense of finite dimensional distributions to

$$\lim_{M\to\infty} \frac{1}{M^{1/2}}\sum_{m=1}^{M}(Y^{(i)}(j) - E\{Y^{(i)}(j)\}) \stackrel{d}{=} G_H(j),\ t \ge 0 \tag{10}$$

where $G_H(t)$ represents a stationary Gaussian process whose covariance function has the form $r(k) \sim ck^{2H-2}$.

Proof: From the Central Limit Theorem, the limiting process on the left hand side has a Gaussian distribution with zero mean and a covariance function equal to the covariance function of the superposed increment process. Now, from the fitting process of Subsection B we already have

$$cov\{Y_M\} \approx \psi_{cov} k^{-\beta} = \psi_{cov} k^{2H-2} \tag{11}$$

where the approximation becomes closer as the number of MMPPs increases and equality is achieved as $M \to \infty$. Also, the superposed increment process is stationary since the individual superposed processes are stationary and the lemma is thus proved.

Theorem 1: As $M \to \infty$ and $T \to \infty$, the aggregated cumulative packet arrival process $\{W_M(Tt), t \ge 0\}$ converges in the sense of finite dimensional distributions to

$$\lim_{T\to\infty}\lim_{M\to\infty} \frac{1}{T^H M^{1/2}}\left(W_M(Tt) - \sum_{i=1}^{M}\frac{\lambda_i\mu_i^1 t}{\mu_i^1+\mu_i^2}\right) \stackrel{d}{=} \sqrt{\frac{\psi_{cov}}{H(2H-1)}}B_H(t) \tag{12}$$

where $B_H(t)$ denotes Fractional Brownian Motion.

Proof: From Lemma 1, we know that the increment process of the aggregate packet process converges to a stationary Gaussian process with a covariance function of the form $r(k) \sim ck^{2H-2}$ as $M \to \infty$. Using Theorem 7.2.11, p. 337 of [13], for a stationary Gaussian sequence $\{G(j), j = \cdots, -1, 0, 1, \cdots\}$ with zero mean and autocovaiance function satisfying $r(k) \sim ck^{2H-2}$ with $1/2 < H < 1$, the finite dimensional distributions of $T^{-H}\sum_{j=1}^{Tt} G(j)$, $0 \le t \le 1$ converge to

$$\lim_{T\to\infty}\frac{1}{T^H}\sum_{j=1}^{Tt} G(j) \stackrel{d}{=} H^{-1/2}(2H-1)^{-1/2}c^{1/2}B_H(t) \tag{13}$$

Hence we can conclude that the aggregated increment process, and thus the aggregated arrival process, converges to fractional Brownian motion as $T \to \infty$, $M \to \infty$ with $c = \psi_{cov}$ in Equation (13).

III Fractional-ARIMA Traffic Models

Long-range dependent traffic models based on *fractional autoregressive integrated moving average* (FARIMA) processes are generalizations of the well-known $ARIMA(p, d, q)$ models of Box-Jenkins [4]. Fractional ARIMA processes allow generalizations of the parameter d to take on non-integer values and were proposed in [8]. These processes are asymptotically second order self-similar with self-similarity parameter $d + 1/2$ with $0 < d < 1/2$. We now describe fractional ARIMA processes in greater detail and show their convergence to fractional Brownian motion.

A Preliminaries

The $FARIMA(0,d,0)$ can be heuristically derived as the discrete-time analogue of continuous time fractional Gaussian noise. The fractional difference operator ∇^d is defined as the binomial series

$$\begin{aligned}\nabla^d &= (1-B)^d = \sum_{k=0}^{\infty}\binom{d}{k}(-B)^k \\ &= 1 - dB - \frac{d}{2}(1-d)B^2 - \frac{d}{6}(1-d)(2-d)B^3\cdots \\ &= \sum_{j=0}^{\infty} b_j(-d)B^j \qquad (14)\end{aligned}$$

where B is the backward shift operator defined as $B^j x_t = x_{t-j}$ and

$$b_j(-d) = \prod_{k=1}^{j}\frac{k+d-1}{k} = \frac{\Gamma(j+d)}{\Gamma(d)\Gamma(j+1)}, \quad j = 1,2,\cdots \qquad (15)$$

where Γ denotes the gamma function. The $FARIMA(0,d,0)$ is formally defined as a discrete-time stochastic process $\{x_t\}$ which is represented as

$$\nabla^d x_t = a_t \qquad (16)$$

or equivalently,

$$x_t = \sum_{j=0}^{\infty} b_j(-d)a_{t-j}, \quad t = \cdots,-1,0,1,\cdots \qquad (17)$$

where the operator ∇^d is defined in Equation (14) and the noise process a_t consists of i.i.d. random variables with zero mean and finite variance [8]. In a more general case, though, the innovations a_t may be from stable distributions with infinite variance. The generalized $FARIMA(p,d,q)$ process where p and q are integers and d real is then defined as the stochastic process $\{y_t\}$ with

$$\phi(B)\nabla^d y_t = \theta(B)a_t \qquad (18)$$

where ∇^d is defined in Equation (14), $\phi(B) = 1 - \phi_1 B - \cdots - \phi_p B^p$, $\theta(B) = 1 - \theta_1 B - \cdots - \theta_q B^q$ and the coefficients $\phi_1 \cdots \phi_p$ and $\theta_1 \cdots \theta_q$ are constants. $FARIMA(p,d,q)$ processes are capable of modeling both short and long-range dependence in traffic models since the effect of d on distant samples decays hyperbolically as the lag increases while the effects of p and q decay exponentially. Thus a $FARIMA(p,d,q)$ process is similar to the $FARIMA(0,d,0)$ for observations with large lags.

B Covariance Structure of FARIMA Processes

When $d < 1/2$, the FARIMA process is stationary and has a representation given in Equation (18). The covariance function of a $FARIMA(0,d,0)$ process with zero mean and unit variance Gaussian innovations has the form

$$\begin{aligned}\gamma(k) &= \frac{(-1)^k(-2d)!}{(k-d)!(-k-d)!} \\ &\sim \frac{\Gamma(1-2d)\sin(\pi d)}{\pi}k^{2d-1} \quad \text{as } k\to\infty \qquad (19)\end{aligned}$$

where $-1/2 < d < 1/2$ and the series is stationary [8]. The covariance function of the generalized $FARIMA(p,d,q)$ processes with Gaussian innovations has additional short-term components but follows the same asymptotic relation as the covariance function as $FARIMA(0,d,0)$ processes.

C Convergence to Fractional Brownian Motion

Let x_i, $i \geq 1$ be a $FARIMA(p,d,q)$ processes with Gaussian innovations defined by Equation (18) with a covariance function as in Equation (19). We define the process $W_M(Tt)$ as the aggregated count process of the series x_i, $i \geq 1$,

$$W_M(Tt) = \sum_{i=1}^{\lfloor Tt\rfloor}\sum_{j=0}^{M} b_j(-d)a_{i-j} \qquad 0 \leq t \leq 1 \qquad (20)$$

where the inside sum becomes x_i as $M \to \infty$. We now show that this aggregated process converges to fractional Brownian motion in the sense of finite dimensional distributions.

Lemma 2: As $M \to \infty$, the discrete time $FARIMA(p,d,q)$ process x_i with Gaussian innovations converges in the sense of finite dimensional distributions to

$$\lim_{M\to\infty}\frac{1}{M^{1/2}}\sum_{j=0}^{M} b_j(-d)a_{i-j} \overset{d}{=} G_H(i) \qquad (21)$$

where $G_H(j)$ represents a stationary Gaussian process whose covariance function has the form $r(k) \sim ck^{2H-2}$ and $0 < d < 1/2$.

Proof: Using the Central Limit Theorem, it is easy to see that the limiting distribution of x_i is Gaussian with zero mean and a covariance function as that of x_i. The zero mean of the limiting Gaussian process follows from the fact that the innovations are zero mean and the process is stationary for $d < 1/2$ [8]. Also, since the covariance function of x_i

$$\begin{aligned}\gamma(k) &\sim \frac{\Gamma(1-2d)\sin(\pi d)}{\pi}k^{2d-1} \qquad (22) \\ &= \frac{-\Gamma(2-2H)\cos(\pi H)}{\pi}k^{2H-2}\end{aligned}$$

where $H = d + 1/2$, has the same form as ck^{2H-2} with $c = (-\Gamma(2-2H)\cos(\pi H))/\pi$ as $k \to \infty$, the lemma is thus proved.

Theorem 2: As $M \to \infty$ and $T \to \infty$, the process represented by the aggregated Gaussian $FARIMA(p,d,q)$

process, $\{W_M(Tt),\, 0 \leq t \leq 1\}$, converges in the sense of finite dimensional distributions to

$$\lim_{T\to\infty} \frac{1}{T^H M^{1/2}} W_M(Tt) \stackrel{d}{=} \sqrt{\frac{-\Gamma(2-2H)\cos(\pi H)}{\pi H(2H-1)}} B_H(t) \tag{23}$$

where $0 < d < 1/2$ and $B_H(t)$ represents fractional Brownian motion.

Proof: The $FARIMA(p,d,q)$ process $x_i,\ i \geq 1$, is the increment process for the aggregate process $W_M(Tt)$. From lemma 2, the limiting distribution of x_i corresponds to a stationary Gaussian sequence with zero mean and a covariance function of the form $\gamma(k) \sim ck^{2h-2}$ as $k \to \infty$. Then, using Theorem 7.2.11, p. 337 of [13], for $1/2 < H < 1$, the finite dimensional distributions of $T^{-H}\sum_{j=1}^{\lfloor Tt \rfloor} x_i,\ 0 \leq t \leq 1$ converge to

$$\lim_{T\to\infty} \frac{1}{T^{-H}} \sum_{i=1}^{\lfloor Tt \rfloor} x_i \stackrel{d}{=} H^{-1/2}(2H-1)^{-1/2} c^{1/2} B_H(t) \tag{24}$$

Hence we can conclude that the aggregated $FARIMA(p,d,q)$ process converges to fractional Brownian motion as $T \to \infty$ with $c = (-\Gamma(2-2H)\cos(\pi H))/\pi$ in Equation (24).

IV Weak Convergence to Fractional Brownian Motion

We proved the convergence of the finite dimensional distributions of MMPP and fractional ARIMA based long range dependent models to fractional Brownian motion in the previous sections. We now propose the following theorem to prove the weak convergence of these models to fractional Brownian motion.

Theorem 3: As $M \to \infty$ and $T \to \infty$, the limiting processes $\{W_M(Tt), 0 < t < 1\}$, as defined in Equations (9) and (20), converges weakly in the space of continuous functions C to

$$\lim_{T\to\infty}\lim_{M\to\infty} \frac{1}{T^H M^{1/2}} \left(W_M(Tt) - \mathrm{E}\{W_M(Tt)\}\right) = H^{-1/2}(2H-1)^{-1/2} c^{1/2} B_H(t) \tag{25}$$

where $B_H(t)$ represents fractional Brownian motion and $c = \psi_{cov}$ for the MMPP model and $c = (-\Gamma(2-2H)\cos(\pi H))/\pi$ for the fractional ARIMA model.

Proof: To prove the weak convergence to fractional Brownian motion of these models in the space of continuous functions C, following Theorem 8.1, p. 54 of [3], it is sufficient to prove their tightness. From Theorem 12.3, p. 95 of [3], a process $X(t)$ is tight if, for all t_1, $t_2 \geq 0$ and some $\gamma \geq 0$ and $\alpha > 1$,

$$E\{|X(t_2) - X(t_1)|^\gamma\} \leq |F(t_2) - F(t_1)|^\alpha \tag{26}$$

where F is a continuous function on $[0,1]$. For our case, $X(t)$ is the limit in Equations (12) and (23) after letting $T \to \infty$ and $M \to \infty$ which we denote by $Y(Tt)$. We choose $\gamma = 2$ and shall now show that Equation (26) holds for $F(t) = Ct$ with some $\alpha > 1$ and a constant C. We know that $X(t)$ is Gaussian with a variance function $\gamma(t)$ of the form $\gamma(t) = \sigma^2 t^{2H}$ and has stationary increments. Then

$$\begin{aligned} E\{|Y(Tt_2) - Y(Tt_1)|^2\} &= T^{-2H}\gamma(T(t_2-t_1)) \\ &= T^{-2H}\sigma^2(T(t_2-t_1))^{2H} \\ &= \sigma^2(t_2-t_1)^{2H} \end{aligned} \tag{27}$$

which is of the form $C(t_2-t_1)^\alpha$ with $C = \sigma^2$ and $\alpha = 2H$. Since $\alpha > 1$ for $0.5 < H < 1$, this establishes tightness and consequently the weak convergence.

V Numerical Results

In this section we present the buffer overflow probabilities of queues fed with long-range dependent traffic generated by FARIMA and fractional Brownian motion sources. Since MMPP and FARIMA based long-range dependent sources weakly converge to fractional Brownian motion in the space of continuous functions, their queueing behavior should show similar characteristics and have a Weibullian shape. Also, for large enough M and T and when the parameters of the three processes are matched, their queueing behaviors should be identical.

In Figure 1 we compare the overflow probability of a queue fed with FARIMA and fractional Brownian motion for utilization factor of 0.7, 0.8 and 0.9. The probabilities were generated by feeding traces with Hurst parameters of 0.8 and 0.9 to a fluid queue with constant service rate. As can be seen, the overflow probabilities for both the processes are very close for most buffer sizes and there is a slight deviation at large buffer sizes. This is due to the fact that the FARIMA traces were not as long as the fractional Brownian motion traces. We were unable to provide results for the MMPP based sources as the state complexity of the superposed process increases exponentially as M increases making it extremely difficult to generate traffic traces for even moderately large M.

VI Conclusion and Discussions

In this paper, we presented convergence results for Markovian and fractional ARIMA based long-range dependent sources and first showed that their limiting distribution converges to fractional Brownian motion in the sense of finite dimensional distributions. We then proved the weak convergence of these models to fractional Brownian motion in the space of continuous functions. These results not only allow us to estimate the performance characteristics of network elements fed with traffic from the sources considered in through already existing tools but also lead to parsimonious and physically meaningful models.

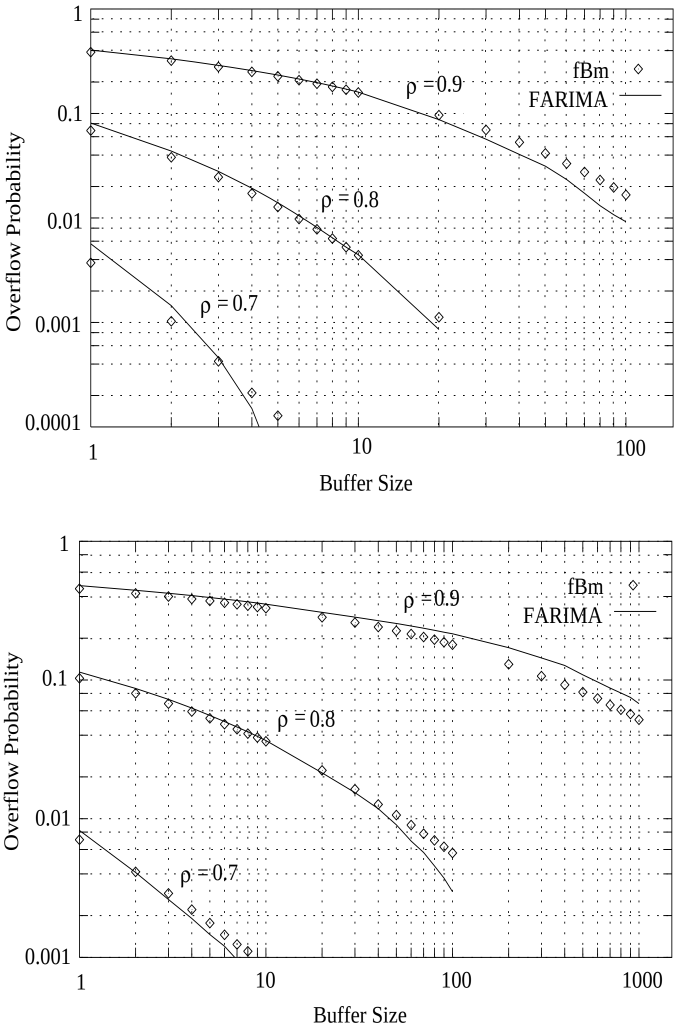

Figure 1: Overflow probabilities in a queue fed with FARIMA and fractional Brownian motion with Hurst parameters of 0.8 and 0.9. Each graph shows the results for utilization factors of 0.7, 0.8 and 0.9.

The weak convergence of the traffic models considered to fractional Brownian motion suggests that performance of network elements with these processes as the workload model will have the same characteristics as that for fractional Brownian motion. More specifically, for example, the tails of queues fed with traffic generated by these two processes also have a Weibullian shape. In fact the weak convergence implies exactly identical behavior of queues fed with either of the three processes, as was numerically verified in Section V.

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