Modeling and Analysis of Energy Harvesting Nodes in Wireless Sensor Networks

Alireza Seyedi
Department of Electrical and Computer Engineering
University of Rochester

Biplab Sikdar
Electrical, Computers and Systems Engineering Department
Rensselaer Polytechnic Institute

***Abstract*—A Markov based unified model for energy harvesting nodes in wireless sensor networks is proposed. Using the presented model, the probability of event loss due to energy run out as well as an analytical vulnerability metric, namely average time to energy run-out, are derived. The results provide insight into the performance of energy harvesting nodes in wireless sensor networks as well as design requirements for such nodes. The proposed vulnerability metric can be used in the various energy aware (harvesting aware) techniques at different protocol layers.**

I. Introduction

The energy supply for the sensor nodes is one of the major issues in the development and widespread deployment of WSN technology[1][2]. The current state-of-the-art battery technology does not provide sufficient energy density to allow the implementation of WSN nodes with long lifetime, low cost, and small form factor.

The most promising approach to address the energy supply problem for WSNs is energy harvesting (or energy scavenging) [2]-[11]. In this approach the sensors are equipped with an energy harvesting device that collects energy from ambient sources such as motion, light, and heat. Energy harvesting expands the design space of communication systems into a new dimension. Traditionally, either it is assumed that the energy supply is unlimited, or that the energy supply is monotonically decreasing with a fixed initial value. Since energy harvesting sensors can replenish their supply of energy, they require a much more sophisticated energy model. Thus, in addition to the previously considered factors such as channel models and traffic models, *energy models* must also be considered as an essential factor in the design of communication systems.

Many energy aware (harvesting aware) communication techniques have been proposed in the literature. Such techniques require a vulnerability metric or cost function for each node to maximize network lifetime and utility. For battery powered sensor nodes this vulnerability metric is directly related to the remaining energy level in the node battery. When energy harvesting is used, however, the vulnerability metric must reflect not only the remaining energy level, but also the harvesting state of the device. The existing literature often use a heuristically defined vulnerability metric [12].

In this paper we first provide a unified model that combines the energy model and the traffic model. Using this model which describes the state of the system by including both the harvesting state as well as the remaining energy supply of the node, we provide an analysis of the Loss Probability due to Energy Run-Out (LPERO). Furthermore, we provide an analytical vulnerability metric, namely Average Time to Energy Run-Out (ATERO).

II. System Model

A. Energy and Traffic Models

We model the energy harvesting process with a two state Markov chain. That is, we assume that the harvesting device is in one of the two activity states, namely *active* and *inactive*. This model can be applied to many different forms of energy harvesting, such as solar energy harvesting in an environmental monitoring sensor network or motion energy harvesting in a body sensor network. We assume that at the end of each time slot the device will go from *active* to *inactive* with probability r, and from *inactive* to *active* with probability w. Consequently, the probabilities that the device stays in the *active* or *inactive* states are $1-r$ and $1-w$, respectively. Furthermore, we assume that the energy harvesting device will harvest energy with an average rate (power) of ρ_a in the *active* state and does not harvest any energy in the *inactive* state. Given this model, it can easily be shown that the steady state probability that the device is in the *active* and *inactive* states are $\mu_w = \frac{w}{r+w}$ and $\mu_r = \frac{r}{r+w}$, respectively. Therefore, the average expected harvested power will be given by $\rho = \frac{w}{r+w}\rho_a$.

We consider a simple traffic model. That is, we assume that in each time slot, an event occurs with probability p. Furthermore, we assume that each event has a total energy cost of E_0, which includes the energy consumed by the sensor, the energy consumed for signal processing as well as the transmission or reception of the related data.

B. Overall System Model

To combine the energy and traffic models described above, we consider the time unit to be equal to the length of the time required to harvest enough energy for one event, given that the device is in the *active* state. That is, we define the time unit to be $T = E_0/\rho_a$. Moreover, we assume that the total battery capacity for the node is equal to $B = (N-1)E_0$.

With this definition of the time unit, we can consider four different cases for each time slot: (i) If the device is in the *inactive* state and an event occurs, the energy level in the node

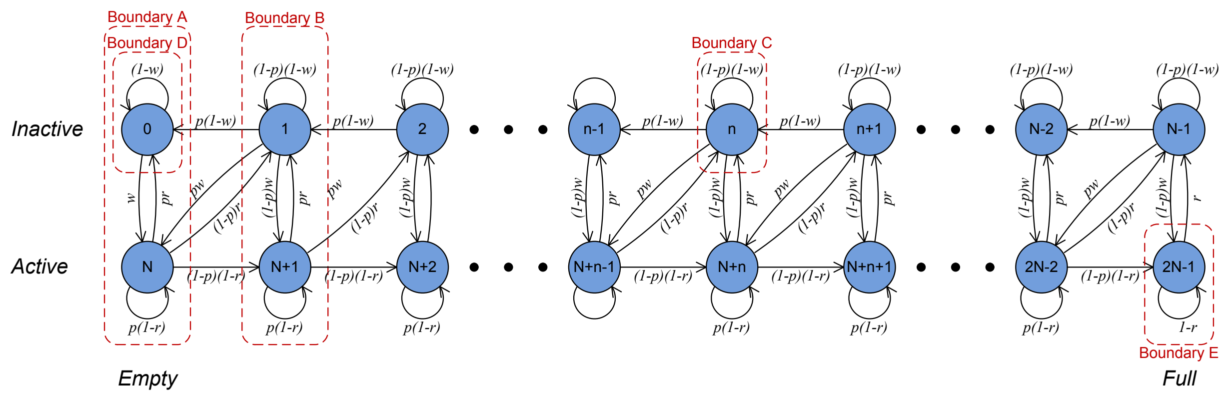

Fig. 1. Markov chain model combining energy model and traffic models.

will decrease by E_0, unless the node has already ran out of energy. (ii) If the device is in the *inactive* state and an event does not occur, the energy level in the node will not change. (iii) If the device is in the *active* state and an event occurs, the energy level in the node will not change. (iv) If the device is in the *active* state and an event does not occur, the energy level in the node will increase by E_0, unless it has reached its maximum, B (the battery is full).

We can then model the state of the node using a $2N$ state Markov chain. The state of the node will represent its energy harvesting state (*i.e. active* or *inactive*) as well as the amount of energy stored in the battery. Such a Markov chain is depicted in Figure 1. In this model, states 0 through $N-1$ represent the case where the device is in the *inactive* state. The device is in the *active* state if the system is in states N through $2N-1$. Furthermore, the amount of energy stored in the battery is given by $E = (n \bmod N)E_0$, where n is the state number. In other words, the battery is empty in states 0 and N, and is completely full in states $N-1$ and $2N-1$.

The components of the transition matrix $\mathbf{P}$ for this Markov chain are given by

$$P_{i,j} = \begin{cases} (1-p)(1-w) & j = i \\ p(1-w) & j = i-1 \\ (1-p)w & j = i+N \\ pw & j = i+N-1 \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

and

$$P_{i,j} = \begin{cases} p(1-r) & j = i \\ (1-p)(1-r) & j = i+1 \\ pr & j = i-N \\ (1-p)r & j = i-N+1 \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

for $i = 1, ..., 2N-2$. Furthermore, we have $P_{0,0} = 1 - w$, $P_{0,N} = w$, and $P_{0,j} = 0$ when $j \neq 0$ or N. Similarly, $P_{2N-1,2N-1} = 1-r$, $P_{2N-1,N-1} = r$, and $P_{0,j} = 0$ when $j \neq 2N-1$ or $N-1$.

III. Loss Probability Due to Energy Run-Out

We are interested in finding the average probability that an event is lost (*i.e.* is not sensed and reported), due to lack of sufficient energy in the node. To find this probability, we note that an event is lost if and only if the event occurs when the node is in state 0. Note that when the node is in state N it will collect sufficient energy during one time slot for the event. When the node is in all other states, it has sufficient energy stored in its battery. Therefore, we can write the probability of event loss by

$$p_L = \pi_0 \tag{3}$$

where π_n denotes the steady state probability that the Markov chain is in state n. Note that since this Markov chain is of finite length, irreducible and non periodic, it has a steady state.

The steady state probabilities for all states can be obtained using the eigen analysis of the transition matrix, $\mathbf{P}$. That is, if the eigen decomposition of the transition matrix is given by

$$\mathbf{P} = \mathbf{U}^{\dagger}\mathbf{\Lambda}\mathbf{U} \tag{4}$$

then the columns of $\mathbf{U}$, $\mathbf{u_k}, k = 0, ..., 2N-1$, are the eigen vectors of $\mathbf{P}$ and the components of the diagonal matrix $\mathbf{\Lambda}$, $\lambda_k, k = 0, ..., 2N-1$, are its eigen values. Then the steady state probabilities $\boldsymbol{\pi} = [\pi_0, ..., \pi_{2N-1}]^T$ will be given by the eigen vector corresponding to an eigen value of unity. That is,$\boldsymbol{\pi} = \mathbf{u}_{k_0}$, where $\lambda_{k_0} = 1$.

While this method provides a general solution for the steady state probabilities, the computation of eigen analysis of the matrix $\mathbf{P}$ quickly grows with N. The particular structure of this Markov chain, however, allows us to derive a closed form solution for the steady state probability π_0. To solve for $\boldsymbol{\pi}$, we write the equilibrium equations for different combination of states.

Considering a boundary containing the states 0 and N (Figure 1, Boundary A), the equilibrium equation is

$$pw\pi_1 + p(1-w)\pi_1 - (1-p)(1-r)\pi_N - (1-p)r\pi_N = 0 \tag{5}$$

or

$$p\pi_1 = (1-p)\pi_N. \tag{6}$$

Also, the equilibrium equation for a boundary around states 1 and $N+1$ (Figure 1, Boundary B) results in

$$p\pi_2 + (1-p)\pi_N - p\pi_1 - (1-p)\pi_{N+1} = 0. \tag{7}$$

Substituting (6) in (7) gives

$$p\pi_2 \quad = \quad (1-p)\pi_{N+1}. \tag{8}$$

By continuing this procedure for states n and $N+n$ for $n=1,...,N-1$, we can conclude that

$$\pi_n = \alpha \pi_{N+n-1} \tag{9}$$

where $\alpha = \frac{1-p}{p}$. The equilibrium equation for a boundary around state n, where $1 \le n \le N-2$ (Figure 1, Boundary C) results in

$$\begin{aligned}[p(1-w)+pw+(1-p)w]\pi_n - p(1-w)\pi_{n+1} \\ -pr\pi_{N+n} - (1-p)r\pi_{N+n-1} = 0\end{aligned} \tag{10}$$

By substituting (9) in (10) we get

$$\begin{aligned}[p+(1-p)w-pr]\,\pi_n \\ -[(1-p)(1-w)+pr]\pi_{N+n} = 0\end{aligned} \tag{11}$$

or

$$\pi_{N+n} = \beta \pi_n \tag{12}$$

where $\beta = \frac{p+(1-p)w-pr}{(1-p)(1-w)+pr}$. Also, the equilibrium equation for a boundary around state 0 (Figure 1, Boundary D) results in

$$w\pi_0 - pr\pi_N + p(1-w)\pi_1 = 0 \tag{13}$$

which using (6) becomes

$$w\pi_0 - [pr + (1-p)(1-w)]\,\pi_N = 0 \tag{14}$$

or

$$\pi_N = \gamma \pi_0 \tag{15}$$

where $\gamma = \frac{w}{pr+(1-p)(1-w)}$. Finally, the equilibrium equations for a boundary around state $2N-1$ (Figure 1, Boundary E) result in

$$r\pi_{2N-1} - (1-p)(1-r)\pi_{2N-2} - (1-p)w\pi_{N-1} = 0 \tag{16}$$

which using (6) becomes

$$r\pi_{2N-1} = [p(1-r)+(1-p)w]\,\pi_{N-1} \tag{17}$$

or

$$\pi_{2N-1} = \delta \pi_{N-1} \tag{18}$$

where $\delta = \frac{p(1-r)+(1-p)w}{r}$. By combining (9), (12), (15), and (18), we can have

$$\pi_n = \alpha^n \beta^{n-1} \gamma \pi_0 \tag{19}$$

and

$$\pi_{N+n-1} = \alpha^{n-1} \beta^{n-1} \gamma \pi_0 \tag{20}$$

for $1 \le n \le N-1$, and

$$\pi_{2N-1} = \alpha^{N-1} \beta^{N-2} \gamma \delta \pi_0. \tag{21}$$

Now we apply the constraint $\sum_{n=0}^{2N-1} \pi_n = 1$, or

$$\begin{aligned}\pi_0 + \sum_{n=1}^{N-1} \alpha^n \beta^{n-1} \gamma \pi_0 + \sum_{n=1}^{N-1} \alpha^{n-1} \beta^{n-1} \gamma \pi_0 \\ +\delta \alpha^{N-1} \beta^{N-2} \gamma \pi_0 = 1\end{aligned} \tag{22}$$

yields

$$\pi_0 = \frac{1}{1+\gamma(\alpha+1)\frac{1-\alpha^{N-1}\beta^{N-1}}{1-\alpha\beta} + \delta\alpha^{N-1}\beta^{N-2}\gamma}. \tag{23}$$

IV. Average Time to Run-Out

We propose the use of the Average Time to Run-Out (ATERO) as a vulnerability metric for each node. We define $T_{n\to k}$ as the average number of steps that is required to visit state k for the first time, if we start at state n. With this notation, the ATERO for state n is represented by $T_{n\to 0}$.

One method to calculate the ATERO is to modify the Markov chain by making state 0 an absorbing state. That is the modified Markov chain will have a transition matrix, $\mathbf{P}'$, where $P'_{0,0} = 1$, $P_{0,N} = 0$, and $P'_{i,j} = P_{i,j}$ for all other i,j pairs. Then $T_{n\to 0}$ in the original Markov chain is equal to the average time to absorbtion in the modified chain. In general, if the transition matrix is of the form

$$\mathbf{P}' = \begin{bmatrix} \mathbf{I} & \mathbf{0} \\ \mathbf{T} & \mathbf{S} \end{bmatrix} \tag{24}$$

the *fundamental matrix* of the Markov chain is defined as $\mathbf{Q} = (\mathbf{I}-\mathbf{S})^{-1}$. Given this form, the number of times, starting in state i, we expect to visit state j before absorption is the ijth entry of $\mathbf{Q}$. Hence, the total number of steps expected before absorption equals the total number of visits we expect to make to all the non-absorbing states. This is the sum of all the entries in the ith row of $\mathbf{Q}$.

Although this provides a general solution, the calculation of $\mathbf{Q}$ for a large Markov chain is very difficult. However, for the particular Markov chain at hand, we can exploit the structure of the chain to recursively calculate $T_{n\to 0}$. This is done by noting that given a start state $n+1$, where $0 \le n \le N-2$, we must visit state n before absorption. Hence, we have

$$T_{n+1\to 0} = T_{n+1\to n} + T_{n\to 0} \tag{25}$$

Along the same line, given a start state $N+n$, where $1 \le n \le N-1$, before absorption, we must visit state n. Hence

$$T_{N+n\to 0} = T_{N+n\to n} + T_{n\to 0}. \tag{26}$$

Thus, if we calculate $T_{n+1\to n}$ and $T_{N+n\to n}$ we have

$$T_{n\to 0} = \sum_{k=1}^{n} T_{k\to k-1} \tag{27}$$

and

$$T_{N+n\to 0} = T_{N+n\to n} + \sum_{k=1}^{n} T_{k\to k-1} \tag{28}$$

for $0 \le n \le N$.

The value of $T_{2N-1\to N-1}$ can be calculated without recursion. That is, we have

$$T_{2N-1\to N-1} = \sum_{k=0}^{\infty} (k+1)(1-r)^k r = \frac{1}{r} \tag{29}$$

Now consider states n, $n+1$, $N+n$ and $N+n+1$, where

$0 \le n \le N-2$. For $T_{N+n\to n}$ we can write

$$\begin{aligned} T_{N+n\to n} &= \sum_{i=0}^{\infty}(i+1)[p(1-r)]^i pr \\ &+ \sum_{i=0}^{\infty}(i+1+T_{n+1\to n})[p(1-r)]^i(1-p)r \\ &+ \sum_{i=0}^{\infty}(i+1+T_{N+n+1\to n}) \\ &\quad \times[p(1-r)]^i(1-p)(1-r) \\ &= \frac{pr}{(1-p+pr)^2}+\frac{(1-p)r}{(1-p+pr)^2} \\ &+ \frac{(1-p)r}{1-p+pr}T_{n+1\to n}+\frac{(1-p)(1-r)}{(1-p+pr)^2} \\ &+ \frac{(1-p)(1-r)}{1-p+pr}T_{N+n+1\to n} \\ &= \frac{1}{1-p+pr}+\frac{(1-p)r}{1-p+pr}T_{n+1\to n} \\ &+ \frac{(1-p)(1-r)}{1-p+pr}T_{N+n+1\to n} \\ &= a+bT_{n+1\to n}cT_{N+n+1\to n}, \end{aligned} \tag{30}$$

where $a=\frac{1}{1-p+pr}$, $b=\frac{(1-p)r}{1-p+pr}$, and $c=\frac{(1-p)(1-r)}{1-p+pr}$.

Along the same lines, for $T_{n+1\to n}$ we have

$$\begin{aligned} T_{n+1\to n} &= \sum_{i=0}^{\infty}(i+1)[(1-p)(1-w)]^i p(1-w) \\ &+ \sum_{i=0}^{\infty}(i+1+T_{N+n\to n})[(1-p)(1-w)]^i pw \\ &+ \sum_{i=0}^{\infty}(i+1+T_{N+n+1\to n}) \\ &\quad \times[(1-p)(1-w)]^i(1-p)w \\ &= \frac{p(1-w)}{(p+w-pw)^2} \\ &+ \frac{pw}{(p+w-pw)^2}+\frac{pw}{p+w-pw}T_{N+n\to n} \\ &+ \frac{(1-p)w}{(p+w-pw)^2}+\frac{(1-p)w}{p+w-pw}T_{N+n+1\to n} \\ &= \frac{1}{p+w-pw}+\frac{pw}{p+w-pw}T_{N+n\to n} \\ &+ \frac{(1-p)w}{p+w-pw}T_{N+n+1\to n} \\ &= d+eT_{N+n\to n}+fT_{N+n+1\to n}, \end{aligned} \tag{31}$$

where $d=\frac{1}{p+w-pw}$, $e=\frac{pw}{p+w-pw}$, and $f=\frac{(1-p)w}{p+w-pw}$.

Also, for $T_{N+n+1\to n}$ we have

$$T_{N+n+1\to n} = T_{N+n+1\to n+1}+T_{n+1\to n}. \tag{32}$$

By employing (32) in (30) and (31) we can solve for $T_{n+1\to n}$ and $T_{N+n\to n}$ as a linear function of $T_{N+n+1\to n+1}$. That is

$$T_{n+1\to n}=AT_{N+n+1\to n+1}+B \tag{33}$$

and

$$T_{N+n\to n}=CT_{N+n+1\to n+1}+D \tag{34}$$

where $A=\frac{f+ec}{1-f-eb-ec}$, $B=\frac{d+ea}{1-f-eb-ec}$, $C=A(b+c)+c$, and $D=B(b+c)+a$.

Now, by recursively applying (34) we get

$$\begin{aligned} T_{N+n\to n} &= C^{N-1-n}T_{2N-1\to N-1}+D\frac{1-C^{N-1-n}}{1-C} \\ &= C^{N-1-n}\frac{1}{r}+D\frac{1-C^{N-1-n}}{1-C}. \end{aligned} \tag{35}$$

And by applying (33) we get

$$T_{n\to n-1}=AC^{N-1-n}\frac{1}{r}+AD\frac{1-C^{N-1-n}}{1-C}+B. \tag{36}$$

Therefore, using (27) we have

$$\begin{aligned} T_{n\to 0} &= \sum_{k=1}^{n}\left(AC^{N-1-k}\frac{1}{r}+AD\frac{1-C^{N-1-k}}{1-C}+B\right) \\ &= AC^{N-1}\left(\frac{1}{r}-\frac{D}{1-C}\right)\sum_{k=1}^{n}C^{-k} \\ &\quad +n\left(\frac{AD}{1-C}+B\right) \\ &= AC^{N-2}\left(\frac{1}{r}-\frac{D}{1-C}\right)\frac{1-C^{-n}}{1-C^{-1}} \\ &\quad +n\left(\frac{AD}{1-C}+B\right). \end{aligned} \tag{37}$$

Also, $T_{N+n\to 0}$ is given by (26), (35) and (37).

V. Results

Figure 2 compares the LPERO obtained from theory (Eq. (3) and (23)) with those obtained from simulations. We have assumed that $w=0.005$ and $r=0.045$, which means that the device is in the *active* state 10% of the time. Moreover, we have assumed that $N=20$. We observe that the simulation results and the theoretical results match perfectly.

Figure 3 presents the LPERO versus N. Once again, we have assumed that $w=0.005$ and $r=0.045$. We can see that as expected larger battery capacities lead to significant drop in the event loss probability. We observe that to achieve event loss probability of 10^{-5} or better, we would require a battery size of $B\ge 32E_0$ and $B\ge 99E_0$ for $p=0.01$ and $p=0.03$, respectively. Even battery sizes as large as $B\ge 1000E_0$, however, will not be sufficient to reduce the event loss probability near 10^{-5}.

Figure 4 compares the ATERO obtained from theory (Eq. (37) and (26)) with those obtained from simulations. Again, we have assumed that $w=0.005$ and $r=0.045$. Furthermore, we assume that $p=0.09$ which is 10% lower than the average harvesting rate. Moreover, we have assumed that $N=20$. We see that the simulation results and the theoretical results match very closely.

Figures 5 depicts the ATERO for the three cases where $w=0.05$ and $r=0.45$, $w=0.005$ and $r=0.045$, and $w=0.0005$ and $r=0.0045$, respectively. In all three cases,

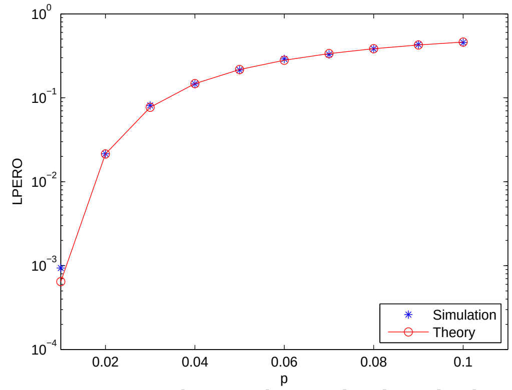

Fig. 2. Comparison between theoretical and simulated LPERO.

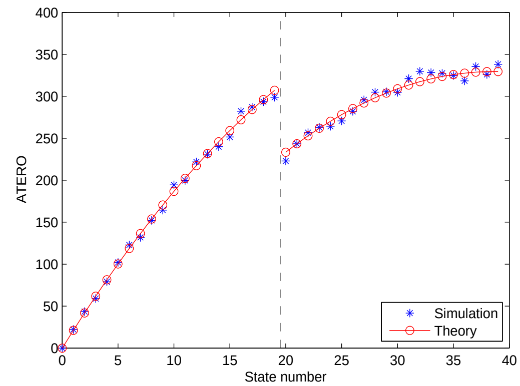

Fig. 4. Comparison of theoretical and simulated ATERO results.

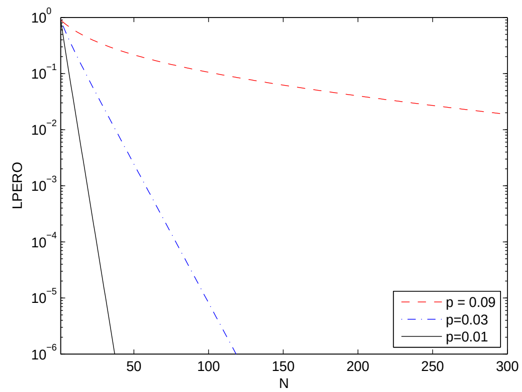

Fig. 3. LPERO as a function of battery size.

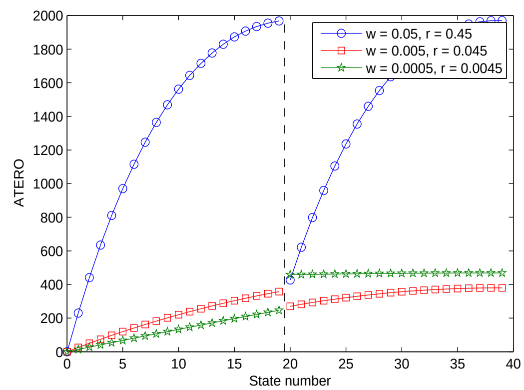

Fig. 5. ATERO for $N = 20$, $p = 0.09$, $w = 0.05$ and $r = 0.45$.

the average harvested power is equal to $\rho = \frac{w}{w+r}\rho_a = 0.1\rho_a$. However, these three cases represent different average lengths of *active* and *inactive* periods. We observe that for large w and r, the states n and $N+n$ have similar ATERO values. This means that the vulnerability of the node can simply be represented by the level of remaining energy in the node and the harvesting state has little information. In contrast, we can see that for small w and r, all of the *active* states have rather large ATERO compared with the *inactive* states. Thus, the vulnerability of the node can be represented only by its harvesting state, with good approximation. In the middle range of the r and w, however, both the remaining energy and the harvesting state information are necessary to represent the relative vulnerability level of each node in the system.

VI. Conclusions

In this paper we have presented a Markov based model for energy harvesting nodes in wireless sensor networks. The presented model considers both the state of the energy harvesting process as well as the remaining energy supply of the node to determine the state of the node. Closed form solutions for the event loss probability and average time to run-out are then derived from the presented model. The results provide insight into the relationship between system parameters such as average harvested power and average traffic rate and maximum battery capacity and give us guidance to set the requirements for energy harvesting nodes in wireless sensor networks. Also, the derived vulnerability metric can be used in various harvesting aware techniques at different protocol layers.

References

[1] I. Akyildiz, W. Su, Y. Sankarasubramaniam and E. Cayirci, "A Survey on Sensor Networks," *IEEE Comm. Mag.*,, pp. 102-114, Aug. 2002.

[2] G. Z. Yang, *Body Sensor Networks*, Springer, London, 2006.

[3] J. A. Paradiso and T. Starner, "Energy scavenging for mobile and wireless electronics," *IEEE Pervasive Computing*, pp. 18-27, Jan.-Mar. 2005.

[4] B. Gyselinckx *et. al.*, "Human++: Emerging Technology for Body Area Networks," in Proc. *IFIP International Conference on Very Large Scale Integration*, pp. 175-180, Oct. 2006.

[5] X. Jiang, J. Polastre and D. Culler, "Perpetual environmentally powered sensor networks," in Proc. *International Symposium on Information Processing in Sensor Networks*, Apr. 2005.

[6] M. Pereyma, "Overview of the Modern State of the Vibration Energy Harvesting Devices," in Proc. *International Conference on Perspective Technologies and Methods in MEMS Design*, pp.107-112, May 2007.

[7] V. Raghunathan, S. Ganeriwal, and M. Srivastava, "Emerging techniques for long lived wireless sensor networks," *IEEE Commun. Mag.*, pp. 108-114, Apr. 2006.

[8] M. Rahimi, H. Shah, G. Sukhatme, J. Heideman and D. Estrin, "Studying the feasibility of energy harvesting in a mobile sensor network," in Proc. *IEEE Conference on Robotics and Automation*, pp. 19-24, Sep. 2003.

[9] E. K. Reilly and P. K. Wright, "Energy Scavenging for Body Sensor Networks," in Proc. *ACM BodyNets'07*, Jun. 2007.

[10] M. Renaud, T. Sterken, P. Fiorini, R. Puers, K. Baert and C. V. Hoof, "Scavenging energy from human body: design of a piezoelectric transducer," in Proc. *The 13th International Conference on Solid-State Sensors, Actuators and Microsystems*, pp. 784-787, Jun. 2005.

[11] N. S. Shenck and J. A. Paradiso, "Energy Scavenging with Shoe-Mounted Piezoelectrics," *IEEE Micro*, pp. 30-42, May/Jun. 2001.

[12] A. Kansal, J. Hsu, M. Srivastava and V. Raghunathan, "Harvesting aware power management for sensor networks," in Proc. *Conference on Design Automation*, Jul. 2006.