Scaling of Spatial Reuse and Saturation Throughput in a Class of MAC Protocols

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***Abstract*—In this paper, we investigate the spatial reuse and saturation throughput of static ad-hoc networks with *unbiased* Medium Access Control (MAC) protocols. Under the stochastic assumptions of our model, we obtain the upper bound on the equivalent saturation throughput of such MAC protocols as a function of node density. We also obtain the scaling properties of the spatial reuse and saturation throughput.**

***Index Terms*—Medium access control, spatial reuse, performance evaluation, ad-hoc networks**

I. Introduction

In many real world ad-hoc network scenarios where the node density is reasonably high, the interference between the nodes becomes the dominant factor affecting the overall network performance. This interference determines the MAC protocol's *spatial reuse* characteristics, i.e., the simultaneous use of the same spectrum in geographically separated locations. In this paper, we propose an analytical framework to evaluate the spatial reuse of a class of MAC protocols in wireless networks and its scaling properties.

Recent research efforts on the performance evaluation of ad-hoc networks usually focus on the problem of the capacity and study its relationship with mobility, connectivity and latency [1], [3]. The classic problem of network capacity in random networks was formulated by Gupta and Kumar in [3] as to find the maximum throughput in both random and arbitrary networks. In this paper, we try to approach the throughput bound from a probabilistic perspective with practical assumptions on the architecture in the MAC layer and develop an analytical framework to derive expressions for the MAC layer saturation throughput. Note that comparison of our results with those of [3] is beyond the scope of this paper since we consider a different underlying model (e.g. single hop MAC layer throughput in this paper versus the end to end throughput of [3], removal of the singularity at the origin etc.).

This paper considers an ad-hoc network with uniform, random node distribution, and a scheduler which works towards maximizing the spatial reuse, provided that the Signal to Interference and Noise Ratio (SINR) constraint is fulfilled at each receiver. The spatial reuse of the network and its scaling are then evaluated using a stochastic model which characterizes the variability and rate of successful transmissions of the MAC traffic at each node. Our results show that in a random access network, with the network getting denser, the aggregate single hop throughput increases at a rate not faster than linear.

The rest of the paper is organized as follows. In Section II we introduce the preliminaries of our model while Section III evaluates the spatial reuse using saturation throughput as the metric. The scaling properties of our model are studied in Section IV and Section V presents the numerical evaluation of our analytical results. Finally, Section VI concludes the paper.

II. Preliminaries

In this section we describe our assumptions and introduce our metric to evaluate the spatial reuse. We assume that all nodes have identical physical layer characteristics, same antenna gains and that there is no statistical dependence between the traffic originating at different nodes.

A. The SINR Model: We use the Physical Model as specified in [3] to characterize the SINR constraint. Let Ω be the set of all nodes on $\mathbb{R}^2$ forming an ad-hoc network. At any given time instant, a number of source-receiver, *S-R pairs* are selected by the MAC scheduler to transmit simultaneously. Suppose the transmitting nodes are $\Omega_S = \{S_i; i \in \Upsilon\}$, and the receiving nodes are $\Omega_R = \{R_i; i \in \Upsilon\}$, with S_i and R_i coupled in the i^{th} S-R pair, for each $i \in \Upsilon$, the set of S-R pairs. Let P_k be the power level chosen by sender S_k. Then the SINR constraint at receiver R_i for a successful reception of the transmission from sender S_i is given by

$$\frac{\frac{P_i}{\|S_i - R_i\|^\alpha}}{\sum_{\substack{k\in\Upsilon \\ k\neq i}} \frac{P_k}{\|S_k - R_i\|^\alpha} + N_i} \geqslant \beta \tag{1}$$

Here α ($\alpha > 2$) is the *path loss exponent*, β is the *SINR threshold*, $\|S_k - R_i\|$ is the distance between sender S_k and receiver R_i and N_i denotes the *ambient noise* around R_i. Assuming all senders use the same transmission power P, we can rewrite Eqn. (1) as

$$\frac{\frac{1}{r_i^\alpha}}{\tau_i + \delta_i} \geqslant \frac{\beta}{\beta + 1} \tag{2}$$

Here $\delta_i = \frac{N_i}{P}$ denotes the normalized ambient noise, and $r_i = \|S_i - R_i\|$ is the one-hop distance for the i^{th} S-R pair. τ_i is defined as the *Aggregate Received Power (ARP)*, which includes both interference power as well as the signal power from its intended sender:

$$\tau_i \triangleq \sum_{k\in\Upsilon} \frac{1}{\|S_k - R_i\|^\alpha} \tag{3}$$

B. A Stochastic Geometric Model for Random Access MAC: We consider a network composed of identical nodes

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randomly and uniformly dispersed on the infinite plane $\mathbb{R}^2$. To characterize the homogeneous distribution pattern of node locations, we use a *Poisson point process*, $\Psi(\lambda_0)$, with λ_0 as *node density* or the expected number of nodes in any region with unit area. Let Φ denote the random distribution of node locations on $\mathbb{R}^2$. Thus $\Phi \sim \Psi(\lambda_0)$, i.e., node locations follow a Poisson point process with density λ_0. The task of a MAC scheduler is to select S-R pairs from the entire node set that satisfy the SINR constraint of Eqn. (2) at each receiver. A MAC scheduler $\Gamma^{(k)}$, representing the k^{th} scheduler in the family of schedulers Θ, can be expressed as a mapping operator: $\Gamma^{(k)} : \Phi \mapsto \{\Phi_S^{(k)}, \Phi_R^{(k)}\},\ \forall k \in \Theta$. In this paper we are particularly interested an *unbiased* MAC schedulers. We define a MAC scheduler to be *unbiased* when the following statement holds true: if node distribution Φ is a Poisson point process, then Φ_S and Φ_R, both determined by the MAC scheduler, are also Poisson point processes. The focus on unbiased MACs is motivated by the fact that important and popular protocols such as Aloha, Slotted Aloha and IEEE 802.11 fall in this category. Since the process resulting from sampling a Poisson process is also a Poisson process, any MAC protocol, such as those mentioned above, where nodes scheduled for transmission are chosen randomly, will satisfy the requirements for unbiasedness. The unbiased property basically assumes the traffic generated by a scheduler is homogeneous after averaging over all possible configurations of the node placement generated by the Poisson model, intuitively representing a reasonable behavior for many random schedulers.

C. Spatial Reuse Metric For an unbiased MAC scheduler $\Gamma^{(k)}$, suppose that $\Phi_S^{(k)} \sim \Psi(\lambda^{(k)})$ and $\Phi_R^{(k)} \sim \Psi(\lambda^{(k)})$, i.e., the random distributions for sender and receiver are both Poisson with the same density $\lambda^{(k)}$, given that node distribution Φ is Poisson with density λ_0. We define the *spatial reuse factor* for MAC scheduler $\Gamma^{(k)}$ as the proportion of nodes that are selected by $\Gamma^{(k)}$ to be a sender/receiver:

$$\eta_k \triangleq \frac{\lambda^{(k)}}{\lambda_0},\ \ \forall k \in \Theta \tag{4}$$

Given node distribution $\Psi(\lambda_0)$, the *spatial reuse factor for the network* is the maximum of spatial reuse factor among all the MAC schedulers:

$$\hat{\eta} = \sup_{k \in \Theta} \eta_k \tag{5}$$

Since $\hat{\eta}\lambda_0$ represents the sender/receiver density associated with the best spatial reuse MAC scheduler, we define the *Equivalent Saturation Throughput*, $\hat{\lambda}$, the achievable number of simultaneous S-R pairs that can be contained in a unit area, as: $\hat{\lambda} = \hat{\eta}\lambda_0$.

III. Analytical Framework for Evaluating the Spatial Reuse

We start our discussion with the characterization of the Aggregate Received Power, τ_i at a node as defined in Eqn. (3). We use a Poisson cloud model wherein nodes are randomly distributed according to a Poisson point process with density λ_0 to characterize the spatial distribution of the nodes. With an unbiased scheduler, senders and receivers also form Poisson clouds with density λ, dropping the superscript k from Eqn. (4). Under the homogeneous assumptions of the Poisson cloud, each node is statistically identical and we can drop the subscript i. In [5] we have shown that the probability density function *PDF* for τ, denoted by $f_T(\tau;\lambda)$ is given by:

$$f_T(\tau;\lambda) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \exp\left[-2\pi\lambda\int_{\varepsilon}^{\infty}\rho\left(1 - e^{\frac{j\omega}{\rho^{\alpha}}}\right)\,d\rho - j\omega\tau\right]d\omega \tag{6}$$

where λ is the sender density and ε is a small positive number to avoid the singularity at the origin of the power attenuation function.

A. Upper Bound for Equivalent Saturation Throughput

For each S-R pair selected by some given MAC scheduler, we consider the distance between them, r, (which is also the one-hop distance since this paper only considers one-hop transmissions controlled by the MAC protocol) to be a random variable. Let $f_R(r;\lambda)$ denote the PDF of r, for a given scheduler with sender density λ. From [4], in a Poisson node cloud with density λ_0, the distance from any node to its nearest neighbor, denoted by r_0, follows the Rayleigh distribution with mean $1/2\sqrt{\lambda_0}$: $f_{R_0}(r_0;\lambda_0) = 2\pi\lambda_0 r_0 e^{-\pi\lambda_0 r_0^2}$. Using this result, it can then easily be shown that the cumulative distribution function *CDF* for the one-hop distance r satisfies:

$$F_R(r;\lambda) \leqslant 1 - e^{-\pi\lambda r^2}. \tag{7}$$

To characterize λ, we first obtain the posterior probability that an arbitrary non-sender node picked from the node cloud, after the scheduling is done, is an *eligible* receiver. For a non-sender to be an eligible receiver, it has to satisfy the SINR constraint in Eqn. (2) with respect to its sender. We use $\theta(\lambda,\delta)$ to denote the probability characterizing this *SINR Eligibility* at each non-sender. Here we assume that the ambient noise in Eqn. (2) takes a constant value δ at each node and that there is no statistical dependence between traffic originating at different nodes.

Proposition 1: The probability associated with the SINR Eligibility any non-sender satisfies:

$$\theta(\lambda,\delta) \leqslant \int_0^{\infty} f_T(\tau;\lambda)\left(1 - \exp\left[-\pi\lambda\left(\frac{\beta+1}{\beta}\frac{1}{\tau+\delta}\right)^{\frac{2}{\alpha}}\right]\right)d\tau \tag{8}$$

Proof: Given the Aggregate Received Power τ, the SINR constraint in Eqn. (2) can be rewritten as $r \leqslant \left(\frac{\beta+1}{\beta}\frac{1}{\tau+\delta}\right)^{\frac{1}{\alpha}}$. Since τ solely depends on the density of the sender cloud, it is independent of r. Hence the SINR Eligibility at each non-sender can be represented as the total probability in terms of the sum of conditional probabilities with regard to τ. Then

$$\theta(\lambda,\delta) = \int_0^{\infty} \mathbb{P}\left\{R \leqslant \left(\frac{\beta+1}{\beta}\frac{1}{\tau+\delta}\right)^{\frac{1}{\alpha}}\ \mid\ \tau\right\} f_T(\tau;\lambda)\,d\tau \tag{9}$$

where R denotes a random variable representing the one-hop distance. The cumulative probability $\mathbb{P}\{\cdot\}$ can be replaced by the CDF of one-hop distance r. Therefore, Eqn. (9) becomes

$$\theta(\lambda,\delta) = \int_0^\infty F_R\left(\left(\frac{\beta+1}{\beta}\frac{1}{\tau+\delta}\right)^{\frac{1}{\alpha}};\lambda\right) f_T(\tau;\lambda)\,d\tau \leqslant \int_0^\infty \left(1-\exp\left[-\pi\lambda\left(\frac{\beta+1}{\beta(\tau+\delta)}\right)^{\frac{2}{\alpha}}\right]\right) f_T(\tau;\lambda)\,d\tau$$

which follows Eqn. (7) and concludes the proof. ■

To simplify the notation, we define

$$\hat{\theta}(\lambda,\delta) \triangleq \int_0^\infty f_T(\tau;\lambda)\left(1-\exp\left[-\pi\lambda\left(\frac{\beta+1}{\beta(\tau+\delta)}\right)^{\frac{2}{\alpha}}\right]\right) d\tau \tag{10}$$

Proposition 1 provides the upper bound for $\theta(\lambda,\delta)$. We also have the following proposition to give its lower bound.

Proposition 2: In a Poisson cloud, given node density λ_0 and sender density λ, the probability associated with the SINR Eligibility at each non-sender satisfies:

$$\theta(\lambda,\delta) \geqslant \frac{\lambda}{\lambda_0-\lambda} \tag{11}$$

Proof: In any region, the expected number of non-senders that meet their SINR Eligibility should be no less than the expected number of receivers selected by the scheduler. Because the distributions of all nodes and senders follow the Poisson law, the non-senders also form a Poisson cloud with density $\lambda_0-\lambda$, according to the properties of Poisson process. For each non-sender immersed in the sender cloud, its SINR Eligibility is evaluated independently. From the properties of the Poisson distribution, we therefore have:

$$\mathbb{E}\left[\text{SINR-eligible non-senders in unit area}\right] = \theta(\lambda,\delta)(\lambda_0-\lambda) \geqslant \lambda \tag{12}$$

where λ is the expected number of receivers in unit area, according to the MAC scheduler. The result follows. ■

By combining Proposition 1 and 2, we can establish the following results for the upper bound on the equivalent saturation throughput $\hat{\lambda}$ and spatial reuse factor $\hat{\eta}$:

Proposition 3: Given node density λ_0, the upper bound for Equivalent Saturation Throughput $\hat{\lambda}$ can be obtained from

$$\hat{\lambda} \leqslant \frac{\lambda_0\hat{\theta}(\hat{\lambda},\delta)}{1+\hat{\theta}(\hat{\lambda},\delta)} \tag{13}$$

Corollary 1: Given node density λ_0, the upper bound for the spatial reuse factor $\hat{\eta}$ can be obtained from

$$\hat{\eta} \leqslant \frac{\hat{\theta}(\hat{\eta}\lambda_0,\delta)}{1+\hat{\theta}(\hat{\eta}\lambda_0,\delta)} \tag{14}$$

IV. Scaling Properties

In this section we study the scaling properties of our performance metrics starting with the following well-known result for the distance dilation in a Poisson point process.

Lemma 1: In a Poisson cloud with density λ, for any scalar $k \in \mathbb{R}^+$, the PDF for the distance between nearest neighbors has the following scaling property:

$$f_{R_0}(r;k\lambda) = \sqrt{k} f_{R_0}(\sqrt{k}r;\lambda) \tag{15}$$

In Eqn. (15), substituting 1 for λ and then λ for k in Eqn. (15) yields the following corollary:

Corollary 2: In a Poisson cloud with density λ, the PDF for the distance between nearest neighbors satisfies:

$$f_{R_0}(r;\lambda) = \sqrt{\lambda} f_{R_0}(\sqrt{\lambda}r;1) \tag{16}$$

Corollary 2 shows that PDF's with different λ can be normalized, which removes a free variable from the PDF, turning it into a single-variable function. It also suggests a way to compare various measures among Poisson point processes with different densities.

A. Scaling Properties of Aggregate Received Power

Lemma 2: In a Poisson cloud with sender density λ, for any scalar $k \in \mathbb{R}^+$, the PDF of the Aggregate Received Power τ has the following scaling property:

$$f_T(\tau;k\lambda) = k^{-\frac{\alpha}{2}} f_T(k^{-\frac{\alpha}{2}}\tau;\lambda) \tag{17}$$

Proof: Consider the concentric ring model of Figure 1, which was used in [5] to derive the PDF of τ. According to the dilation properties, when we increase the node density by a factor of k, it amounts to shrinking all the distances by $\sqrt{k}$. This is equivalent to elevating the contribution to the Aggregate Received Power from each ring to

$$\zeta_\rho = \frac{1}{\left(\frac{\rho}{\sqrt{k}}\right)^\alpha} = \frac{k^{\frac{\alpha}{2}}}{\rho^\alpha} \tag{18}$$

as compared to the original contribution $\frac{1}{\rho^\alpha}$. Since the Aggregate Received Power τ is the summation of this contribution over all the rings, we can expect that with a scaled density $k\lambda$ the Aggregate Received Power τ is multiplied by a factor of $k^{\frac{\alpha}{2}}$. Therefore, if we define a new random variable $\tilde{T} = k^{\frac{\alpha}{2}}T$ representing the scaled Aggregate Received Power when density is λ, it should have the same probability distribution as the normal T with a scaled density $k\lambda$: $f_{\tilde{T}}(\tau;\lambda) = f_T(\tau;k\lambda)$. Thus its CDF has the form

$$F_{\tilde{T}}(\tau;\lambda) = \mathbb{P}\{\tilde{T} \leqslant \tau\} = \mathbb{P}\{T \leqslant k^{-\frac{\alpha}{2}}\tau\} = F_T(k^{-\frac{\alpha}{2}}\tau;\lambda) \tag{19}$$

Thus we have

$$f_T(\tau;k\lambda) = \frac{dF_{\tilde{T}}(\tau;\lambda)}{d\tau} = k^{-\frac{\alpha}{2}} f_T(k^{-\frac{\alpha}{2}}\tau;\lambda) \tag{20}$$

which concludes the proof. ■

Again, in Eqn. (17), substituting 1 for λ and then λ for k leads to the following corollary, which removes a free variable from the PDF through the normalization of λ.

Corollary 3: In a Poisson cloud with density λ, the PDF of the Aggregate Received Power τ satisfies:

$$f_T(\tau;\lambda) = \lambda^{-\frac{\alpha}{2}} f_T(\lambda^{-\frac{\alpha}{2}}\tau;1) \tag{21}$$

B. Scaling Properties of Spatial Reuse Factor

To obtain the scaling properties of the spatial reuse factor $\hat{\eta}$, we first discuss the properties for the upper bound on the probability of SINR Eligibility, $\hat{\theta}(\lambda,\delta)$, defined in Eqn. (10).

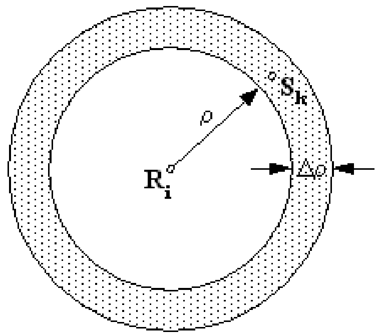

Fig. 1. The concentric ring model. The space is divided into infinite non-overlapping concentric rings centered at R_i, and the superposition of all the rings gives us the entire unbounded region $\mathbb{R}^2$. Each ring has a radius of ρ and an infinitesimal width of $\Delta\rho$, with ρ varying continuously from 0 to ∞. Since $\Delta\rho \to 0$, each ring can hold at most one sender, as per the properties of Poisson processes. The Aggregate Received Power at R_i is the summation of the normalized power level received from the sender inside each ring, if any. In the case that there is a sender inside the ring, this contribution is $\frac{1}{\rho^\alpha}$ per Eqn. (3) and the contribution is 0 in the absence of a sender. Denoting by ζ_ρ the random variable representing the contribution of the ring to the Aggregate Received Power τ at the central node R_i, we have a Bernoulli distribution for ζ_ρ: $\mathbb{P}\{\zeta_\rho = \frac{1}{\rho^\alpha}\} = 2\pi\lambda\rho\Delta\rho + o(\Delta\rho)$ and $\mathbb{P}\{\zeta_\rho = 0\} = 1 - 2\pi\lambda\rho\Delta\rho + o(\Delta\rho)$.

Proposition 4: When ambient noise δ is negligible, $\hat{\theta}(\lambda, \delta)$ does not vary with λ. That is,

$$\frac{\partial\hat{\theta}(\lambda, 0)}{\partial\lambda} = 0 \tag{22}$$

Proof: According to Corollary 3, we have

$$\begin{aligned}\hat{\theta}(\lambda, \delta) &= \int_0^\infty f_T(\tau;\lambda)\left(1 - \exp\left[-\pi\lambda\left(\frac{\beta+1}{\beta}\frac{1}{\tau+\delta}\right)^{\frac{2}{\alpha}}\right]\right)d\tau \\ &= \int_0^\infty \lambda^{-\frac{\alpha}{2}} f_T(\lambda^{-\frac{\alpha}{2}}\tau;1)\left(1 - \exp\left[\frac{-\pi\lambda(\beta+1)^{\frac{2}{\alpha}}}{(\beta(\tau+\delta))^{\frac{2}{\alpha}}}\right]\right)d\tau\end{aligned} \tag{23}$$

We define a new variable $\tilde{\tau} \triangleq \lambda^{-\frac{\alpha}{2}}\tau$ and substitute $\lambda^{\frac{\alpha}{2}}\tilde{\tau}$ for τ in Eqn. (23). Then it becomes

$$\hat{\theta}(\lambda, \delta) = \int_0^\infty f_T(\tilde{\tau};1)\left(1 - \exp\left[\frac{-\pi\lambda(\beta+1)^{\frac{2}{\alpha}}}{(\beta(\lambda^{\frac{\alpha}{2}}\tilde{\tau}+\delta))^{\frac{2}{\alpha}}}\right]\right)d\tilde{\tau} \tag{24}$$

When ambient noise $\delta = 0$, the λ's on the right hand side of Eqn. (24) cancel out. Therefore, $\hat{\theta}(\lambda, \delta)$ is not a function of λ. This concludes the proof. ■

Proposition 4 shows that when ambient noise is negligible, $\hat{\theta}(\lambda, \delta)$ is a constant regardless of the value of λ. We define this constant as:

$$\hat{\theta}_C \triangleq \hat{\theta}(\lambda, 0) = \int_0^\infty f_T(\tau;1)\left(1 - \exp\left[-\pi\left(\frac{\beta+1}{\beta\tau}\right)^{\frac{2}{\alpha}}\right]\right)d\tau \tag{25}$$

As a direct result from Corollary 1, we can obtain the following proposition.

Proposition 5: If we ignore the ambient noise, the spatial reuse factor $\hat{\eta}$ has a constant upper bound of

$$\hat{\eta} \leqslant \frac{\hat{\theta}_C}{1+\hat{\theta}_C} \tag{26}$$

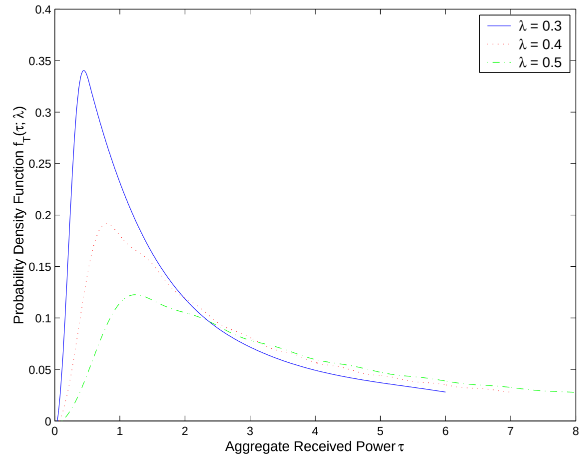

Fig. 2. Probability density function of the Aggregate Received Power τ for different sender densities λ. The x-axis represents the value of random variable τ, and the y-axis represents its probability density $f_T(\tau;\lambda)$.

Proposition 5 reveals that the upper bound for the spatial reuse factor $\hat{\eta}$ is scaling invariant.

V. NUMERICAL RESULTS

In this section, we present a numerical evaluation to visualize the results of our analytical model. We assume that the transmission power level is the same for all nodes, and the path loss exponent α is set to 4 throughout the evaluation. Figure 2 shows the PDF of the Aggregate Received Power τ with different sender densities λ, obtained by numerically evaluating Eqn. (6). We observe that with an increase in λ, the expectation of the Aggregate Received Power grows large while the PDF becomes flat and decays more slowly. This matches our intuition that as the traffic gets more crowded, the interference among S-R pairs would go up.

Figure 3 shows how the upper bound for Equivalent Saturation Throughput $\hat{\lambda}$, our spatial reuse metric, evolves as a function of node density λ_0 for different ambient noise power δ. The curves are obtained by evaluating Eqn. (13) and (24) assuming a SINR threshold $\beta = 10$dB. We see that the bound for $\delta = 0$ is a straight line from the origin. This results from Proposition 5 in that $\hat{\lambda}$ is bounded by $\frac{\hat{\theta}_C}{(1+\hat{\theta}_C)}\lambda_0$ when ambient noise becomes negligible. The slope of the line, which is 0.34 in this figure, is the constant bound for the spatial reuse factor $\hat{\eta}$. Note that this constant bound depends on the SINR threshold β.

From Figure 3 we also observe that when $\delta > 0$ the bound has a non-linear convex shape, which sits under the noise-free linear bound and shifts away as ambient noise δ is aggravated. This can be explained by the fact that ambient noise deteriorates the channel condition and reduces the effective throughput. The convex shape indicates that as nodes become denser, the slope $\hat{\eta}$ slightly decreases, thereby reducing the

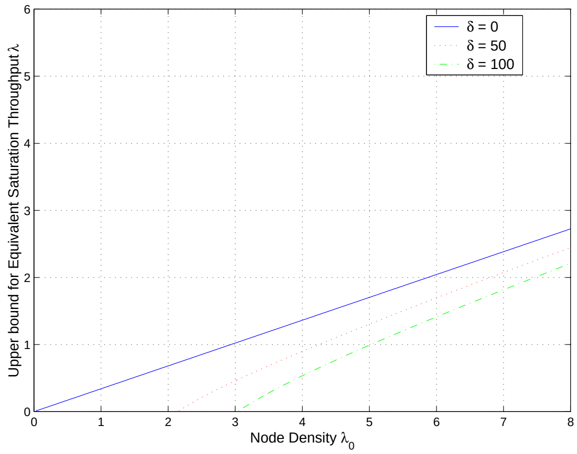

Fig. 3. Upper bound for Equivalent Saturation Throughput $\hat{\lambda}$ as a function of node density λ_0. The curves are shown with different ambient noise power δ. All curves have the same SINR threshold $\beta = 10$dB.

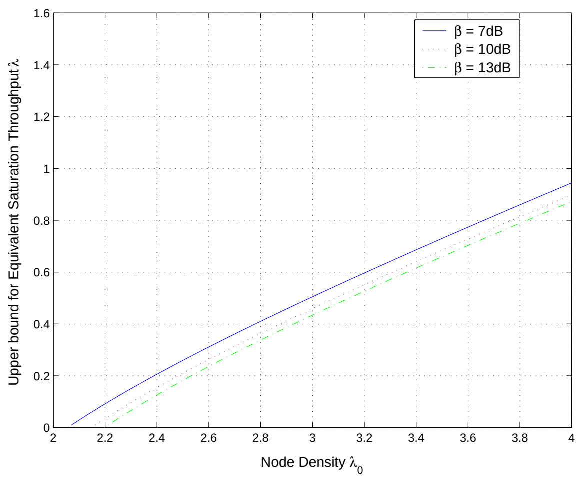

Fig. 4. Upper bound for Equivalent Saturation Throughput $\hat{\lambda}$ as a function of node density λ_0 with different SINR thresholds β, measured in dB. The ambient noise power δ is assumed to be 50.

chance for a node to transmit. It is also seen that these non-linear bounds starts off from a positive offset at $\hat{\lambda} = 0$, which implies that a static ambient noise must be overcome by the transmitter before generating any effective throughput.

Figure 4 demonstrates the upper bounds for $\hat{\lambda}$ with different SINR thresholds β, assuming ambient noise $\delta = 50$. It is seen that the upper bounds slightly drop as β grows, while the shape of the bound is maintained. This suggests that a small SINR threshold in the network can tolerate more interference and thus enhances the spatial reuse. However, a lower SINR threshold would result in greater demands on the hardware.

In Figure 5 we compare the equivalent saturation throughput of IEEE 802.11, obtained through simulations in *ns-2*, with the bound developed in this paper. Since 802.11 stifles all nodes within the transmission radius of nodes of an ongoing transmission, irrespective of the SINR levels, its spatial reuse is much lower than the bound. To show the tightness of our bound, Figure 5 also shows the results for a MAC protocol where each node first selects its nearest neighbor as its pair

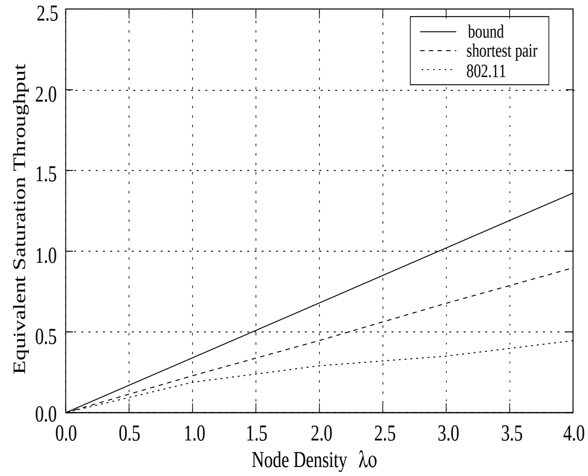

Fig. 5. Equivalent Saturation Throughput $\hat{\lambda}$ for different protocols and its comparison with the theoretical bound as a function of node density λ_0. The SINR thresholds $\beta = 10$dB and the ambient noise power δ is assumed to be 0.

and pairs are then randomly selected for transmission. The difference from the bound for this MAC is about 33%. The performance of an optimal MAC protocol which schedules node pairs with respect to the SINR levels at all nodes will be much closer to the bound, though such a protocol may not be practical.

VI. Conclusions

In this paper, we approach the problem of MAC layer throughput bound from a probabilistic perspective and evaluate it in terms of the spatial reuse. Our results show that in a random access network, with the network getting denser, the one-hop throughput capacity increases at a rate not faster than linear. We also study the scaling properties of our model and the bound on the MAC layer throughput when ambient noise is present, which is shown to have a convex shape.

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