Queueing Analysis of IEEE 802.11 Point Coordination Function

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Abstract. In this paper, we present an analytic model for evaluating the queueing delays at nodes using the IEEE 802.11 Point Coordination Function (PCF) MAC for real time, delay sensitive traffic. We develop a queueing model for each node to obtain closed form expressions for the expected delay which accounts for arbitrary packet sizes, polling rates, channel rates and the order in which the nodes are polled. Our analytical results are verified through simulations.

Keywords: Queueing Analysis, Wireless Networks, MAC protocol

1 Introduction

The IEEE 802.11 MAC [5] has become ubiquitous and gained widespread popularity as a layer-2 protocol for wireless local area and in-home networks. Supporting real-time applications in these environments requires that the MAC layer provide sufficient delay guarantees and the Point Coordination Function has been included in 802.11 to achieve this objective.

The delay characteristics of the 802.11 PCF has been extensively studied using simulations [2, 12, 8]. The effect of different polling strategies on PCF performance [13, 3], improved polling mechanisms for the PCF [4, 14, 9, 15] and the performance of video transmission with PCF [10, 11] have also been investigated in literature. However, these are all simulation studies, and to the best of our knowledge, no detailed queueing or analytic models for 802.11 PCF exists in literature.

This paper proposes a detailed queueing model to evaluate the delays experienced by nodes using 802.11 PCF as the MAC protocol. Our model allows for arbitrary number of users in the network, their packet arrival rates and packet lengths and unidirectional data transfers. The model evaluates the delays as a function of various 802.11 specific parameters like the superframe and beacon lengths facilitating the estimation of the tradeoffs involving the values of these parameters and the system performance.

The rest of the paper is organized as follows. In Section 2 we give a brief overview of the 802.11 PCF. Section 3 presents our delay model for unidirectional data transfer. Section 4 we present the validation results and Section 5 presents the concluding remarks.



Fig. 1. Alternation between CPs and CFPs in a superframe.

2 Background

In addition to the physical layer specifications, the IEEE 802.11 standard [5] specifies two methods for medium access: Distributed Coordination Function (DCF) and the PCF. While DCF uses a distributed, backoff based mechanism for channel access and is not the focus of the paper, in PCF the nodes are polled by a "master" residing within the base station. The channel access mechanism alternates between the DCF and PCF modes when PCF is implemented. The duration of time the DCF is used for channel access is termed the contention period (CP) and the polled duration is called the contention-free period (CFP). The lengths of the CP and the CFP is controlled explicitly by the contention free period repetition interval (CFPri) as shown in Figure 1. We call a CFPri duration where the PCF and DCF alternate a "superframe".

Each CFP begins with a beacon frame and the CFPs occur at a defined repetition rate as determined by the CFPrate parameter. With PCF, the access to the channel is determined centrally by the base station, usually referred to as the Point Coordinator (PC) and provides a contention free transfer service. The PC gains control of the medium at the beginning of the CFP and maintains control for the entire CFP by waiting for a shorter time between transmissions than the stations using the DCF access mode. All stations other than the PC set their NAVs to the CFPMaxDuration at the start of each CFP. The PC transmits a CF-End or CF-End+ACK frame at the end of each CFP and on receiving either of these frames a station resets its NAV. During the CFP, the base station polls the nodes for a *single* pending frame transmission according to a list ordering of their association with the base station, known as the polling list. The PC starts CF transmissions a SIFS interval after the beacon frame by sending a CF-Poll (no data), Data or Data+CF-Poll frame. If a station receives a CF-Poll (no data) frame from the PC, the station can respond to the PC after a SIFS interval with a CF-ACK (no data) or a Data+CF-ACK frame. If the PC receives a Data+CF-ACK frame from a station, it can send a Data+CF-ACK+CF-Poll frame to a different station where the CF-ACK part is used to acknowledge receipt of the previous data frame. If the PC transmits a CF-Poll (no data) frame and the destination station does not have any data to transmit, the station sends a Null Function (no data) frame back to the PC. If the PC fails to receive an ACK for a transmitted data frame, it waits for a PIFS interval and moves on to the next station in the polling list. Figure 2 shows the transmission of frames between the PC and stations.



Fig. 2. PC to station frame transmissions in PCF.

3 Analysis

In this section we present our model to evaluate the delays experienced at the PCF based stations. We assume that an arbitrary number of nodes, M, use the PCF mode to transmit their packets. The packet inter-arrival times at the i^{th} node are assumed to be exponentially distributed with rate λ_i , $1 \leq i \leq M$. We denote the duration of the superframe by T_S and the length of a polling duration by V and the expected length of a packet from the i^{th} polled node by L_i , $1 \leq i \leq M$. Note that we include the lengths of the SIFS and CF-Poll in V and SIFS and CF-ACK in L_i . The utilization of the i^{th} station is denoted by ρ_i . Note that since each polled stations gets to transmit once in every superframe, the service rate of the i^{th} station is $\mu_i = 1/T_S$, $1 \leq i \leq M$. The utilizations are thus given by $\rho_i = \lambda_i/\mu_i = \lambda_i T_S$. In the derivations presented in this paper, we assume that the arrival rates and packet lengths are the same at each node, i.e., $\lambda_i = \lambda$, $\forall i$ and $L_i = L$, $\forall i$ and thus $\rho_i = \rho = \lambda T_S$, $\forall i$.

We now evaluate the expected delay experienced by an arbitrary packet arriving at the i^{th} polled node. We break the analysis into two parts: (1) the delay experienced when the packet arrived at an empty queue and (2) when the arrival occurred at a non empty queue. The probability that an arbitrary arrival finds the queue empty, P[EQ], is given by

$$P[\mathrm{EQ}] = 1 - \rho = 1 - \lambda T_S \tag{1}$$

and the probability that an arbitrary arrival finds the queue busy, P[NEQ], is thus

$$P[\text{NEQ}] = 1 - P[\text{EQ}] = \rho = \lambda T_S.$$
⁽²⁾

Since the arrivals at each queue are independent and the probability that a queue is busy is given by ρ , the number of active queues, j, at any instant of time, out of M queues follows a Binomial distribution and is given by

$$P[j \text{ active}] = \binom{M}{j} \rho^j (1-\rho)^{M-j} \qquad j = 0, 1, \cdots, M$$
(3)

3.1 Arrivals at an Empty Queue

Consider an arrival at the i^{th} polled station whose queue is currently empty and we call this arrival the "tagged arrival". If this station has not yet been polled in the current superframe when the packet arrives, the packet gets served in the current superframe.



Fig. 3. Packet delays when arriving packet finds an empty queue.

Otherwise, the packet gets served in the following superframe. Now, it is well known that with exponential arrivals in a slotted departure system (for example a classical M/D/1 queue), an arrival is equally likely to occur anywhere in a slot or frame [7, 1]. In our case, given that an arrival occurs in a given superframe, the arrival instance is thus uniformly random variable over $[0, T_s]$ relative to the start of the superframe. Consider the case where j of the i - 1 nodes polled before the i^{th} node in a given superframe have data to send. In this case, a period of B + (i-1)V + jL seconds elapse in the superframe before the i^{th} node is polled and B + iV + jL seconds elapse before it has to reply to the poll. Thus if the tagged arrival occurs in this duration, it gets served in this superframe. Otherwise it waits for the next superframe. We now evaluate the probabilities of the associated events and the expected waiting time of the packet.

Since the arrival instant, t, of any packet relative to the start of its superframe is uniformly distributed $(U[0, T_S])$, the probability that the tagged packet arrived at node iin the first B + iV + jL seconds is given by

$$P[t \le B + iV + jL] = \frac{B + iV + jL}{T_S}.$$
(4)

In this case (which we call case C1), the packet waits till the i^{th} node is polled and is then transmitted, as shown in Figure 3. The time the packet waits before it begins service, $X_{i,j,C1}$, is thus $X_{i,j,C1} = B + iV + jL - t$ after which it receives service for another Lseconds before departing the system. We will now characterize the distribution of $X_{i,j,C1}$. The probability distribution function (PDF) of t given that the arrival occurred in the first B + iV + jL seconds of the superframe is given by

$$P[t \le \tau \mid t \le B + iV + jL] = \frac{P[t \le \tau, t \le B + iV + jL]}{P[t \le B + iV + jL]}$$
$$= \frac{\tau}{B + iV + jL}$$
(5)

which is an Uniform distribution in the range 0 to B + iV + jL. Now, note that if a random variable Y is uniformly distributed in the range 0 to a, then the random variable a - Y is also uniformly distributed in the range 0 to a. Following this observation, since the conditional PDF of t is uniformly distributed in the range 0 to B + iV + jL, the

conditional PDF of $X_{i,j,C1} = B + iV + jL - t$ is also an Uniform distribution in the range 0 to B + iV + jL, i.e., U[0, B + iV + jL]. The expected value of $X_{i,j,C1}$ is thus

$$E[X_{i,j,C1}] = E[U[0, B + iV + jL]] = \frac{B + iV + jL}{2}.$$
(6)

In the case where the packet does not arrive in the first B + iV + jL seconds of the superframe (which we call case C2), i.e. t > B + iV + jL, the packet has to wait till the remaining part of the superframe $(T_S - t)$ is over and node *i* is polled in the following round. The PDF of *t* given that the arrival occurred after the first B + iV + jL seconds of the superframe is given by

$$P[t \le \tau \mid t > B + iV + jL] = \frac{P[t \le \tau, t > B + iV + jL]}{P[t \le B + iV + jL]}$$
$$= \frac{\tau - B + iV + jL}{T_S - B + iV + jL}$$
(7)

which is an Uniform distribution in the range B + iV + jL to T_S , i.e., $U[B + iV + jL, T_S]$. Again we note that if a random variable Y follows a Uniform distribution U[a, b], then b - Y is uniformly distributed in the range 0 to b - a, i.e. U[0, b - a]. Thus the duration of the remaining part of the superframe, $T_S - t$, is also uniformly distributed and is $U[0, T_S - B - iV - jL]$.

In the following superframe, if there are k nodes with data to send among the i-1 nodes polled before the i^{th} node, the tagged packet has to wait for B + iV + kL seconds before its service begins. Since the probability that there are k nodes with data among i-1 nodes follows a Binomial distribution as given in Eqn. (3), the probability mass function (pmf) of this waiting time, X_{NR} is given by

$$P[X_{NR} = x] = \begin{cases} \binom{i-1}{k} \rho^k (1-\rho)^{i-k-1} & x = B + iV + kL \\ 0 & \text{otherwise} \end{cases}$$
(8)

with $0 \le k \le i - 1$ and the expected value of X_{NR} given by

$$E[X_{NR}] = B + iV + (i-1)\rho L.$$
(9)

Thus the amount of time, $X_{i,j,C2}$, before the packet begins its service is $X_{i,j,C2} = T_S - t + X_{NR}$. The expected value of $X_{i,j,C2}$ is thus

$$E[X_{i,j,C2}] = E[U[0, T_S - B - iV - jL]] + E[X_{NR}]$$

= $\frac{T_S - B - iV - jL}{2} + B + iV + (i - 1)\rho L$ (10)

To find the expected waiting time in the systems when an arrival occurs at an empty queue given that j of the i-1 nodes before the i^{th} node send data in the current superframe, we combine the waiting times of the above two cases. This expected waiting time, $D_{i,j,EQ}$, is given by

$$D_{i,j,EQ} = E[X_{i,j}] + L \tag{11}$$

with

$$E[X_{i,j}] = E[X_{i,j,C1}]P[C1] + E[X_{i,j,C2}]P[C2]$$
(12)

where $E[X_{i,j,C1}]$ and $E[X_{i,j,C2}]$ are given in Eqns. (6) and (10) respectively and P[C1] and P[C2] are the probabilities that the arrival occurs in the first B + iV + jL seconds of the superframe or not, respectively. As discussed earlier in this section, these are given by

$$P[C1] = \frac{B + iV + jL}{T_S} \qquad P[C2] = 1 - \frac{B + iV + jL}{T_S}$$
(13)

Putting these values in Eqn. (12), $E[X_{i,j}]$ simplifies to

$$E[X_{i,j}] = \frac{T}{2} + \frac{(B+iV+jL)^2}{T_S} - (B+iV+jL) + E[X_{NR}]\frac{T_S - B - iV - jL}{T_S}$$
(14)

which can now be used in Eqn. (11) to obtain $D_{i,j,EQ}$. The expected delay at the i^{th} node, $D_{i,EQ}$ is then obtained by unconditioning Eqn. (11) on j whose pmf given in Eqn. (3). Thus $D_{i,EQ}$ is given by

$$D_{i,EQ} = \sum_{j=0}^{i-1} \left(E[X_{i,j}] + L \right) \binom{i-1}{j} \rho^j (1-\rho)^{i-j-1}$$
$$= \frac{T_S}{2} + \frac{\rho L^2 (i-1)(1-\rho)}{T_S} + L$$
(15)

3.2 Arrivals at a Non-Empty Queue

We now consider the case when an arbitrary arrival to the i^{th} polled node finds the queue non-empty and we denote the number of packets in the queue found by this packet by N_{NQ} . In this case, this tagged arrival has to wait till all preceding arrivals have been served. To calculate the packet's waiting time, we again consider two possible cases: whether the i^{th} node has already been served in the current superframe when the tagged arrival occurs (case C2) or not (case C1). Consider again the case where j of the i - 1 nodes polled before node i have packets to send in the current superframe. Then the probabilities of the events C1 and C2 are given by

$$P[C1] = \frac{B + iV + (j+1)L}{T_S}$$

$$P[C2] = 1 - \frac{B + iV + (j+1)L}{T_S}$$
(16)

where the j + 1 terms comes from the fact that in addition to the j nodes, node i is also transmitting.

In case the i^{th} node has not yet been served when the tagged packet arrives (case C1), one of the N_{NQ} packets currently waiting in the queue at node *i* gets served during this superframe. If we denote the instant of the tagged packet's arrival in the superframe by *t*, it has to wait for $T_S - t$ seconds before the current superframe ends. The tagged packet then has to wait for another $N_{NQ} - 1$ packets to depart, with one departure in one superframe or $(N_{NQ} - 1)T_S$ seconds before the start of the superframe where it receives service. We denote the wait in the final superframe by X_{FR} . Thus the total time before the packet begins service in this case, $X_{i,j,C1}$, is given by $X_{i,j,C1} = T_S - t + (N_{NQ} - 1)T_S + X_{FR}$.

Following the derivation in Eqn. (5), the PDF of t given that the arrival occurred in the first B + iV + (j + 1)L seconds is given by

$$P[t \le \tau \mid t \le B + iV + (j+1)L] = \frac{\tau}{B + iV + (j+1)L}$$
(17)

which is the Uniform distribution U[0, B+iV+(j+1)L]. Again, we note that if a random variable Y is uniformly distributed in the range 0 to a, then the random variable b - Yis also uniformly distributed in the range b - a to b. Thus $T_S - t$ follows the Uniform distribution $U[T_S - B - iV - (j+1)L, T_S]$. To evaluate the distribution of X_{FR} , we note that if there are k nodes with data to send among the i - 1 nodes polled before node i, the packet has to wait for B + iV + kL seconds before its service begins. Since k follows the Binomial distribution of Eqn. (3), the pmf of X_{FR} is given by

$$P[X_{FR} = x] = \begin{cases} \binom{i-1}{k} \rho^k (1-\rho)^{i-k-1} x = B + iV + kL \\ 0 & \text{otherwise} \end{cases}$$
(18)

with $0 \le k \le i - 1$ and the expected value of X_{FR} is

$$E[X_{FR}] = B + iV + (i-1)\rho L.$$
(19)

The expected value of $X_{i,j,C1}$ is thus

$$E[X_{i,j,C1}] = E[T_S - t] + E[(N_{NQ} - 1)T_S] + E[X_{FR}]$$

= $\frac{B + iV + (j+1)L}{2} + (E[N_{NQ}] - 1)T_S + B + iV + (i-1)\rho L$ (20)

In the case where the tagged arrival occurs after the i^{th} node has been served in the current round (case C2), at the end of the current superframe, there are still N_{NQ} packets ahead of the tagged packet. Thus at the end of a further $N_{NQ}T_S$ seconds, the superframe in which the tagged packet gets served starts. The amount of time the tagged packet has to wait in this final round is again denoted by X_{FR} and its pmf and expected values are given in Eqns. (18) and (19) respectively. Thus the total time before the packet begins service in this case, $X_{i,j,C2}$, is given by $X_{i,j,C2} = T_S - t + N_{NQ}T_S + X_{FR}$. Now, following the derivation of Eqn. (7), the PDF of t given that the arrival occurred after the first B + iV + (j + 1)L seconds of the superframe is given by

$$P[t \le \tau \mid t > B + iV + (j+1)L] = \frac{\tau}{T_S - B - iV - (j+1)L}$$

which is the Uniform distribution $U[B+iV+(j+1)L, T_S]$. Thus $T_S - t$ is also uniformly distributed and is $U[0, T_S - B - iV - (j+1)L]$. The expected value of $X_{i,j,C2}$ is thus

$$E[X_{i,j,C2}] = E[T_S - t] + E[N_{NQ}T_S] + E[X_{FR}]$$

= $\frac{T_S - B - iV - (j+1)L}{2} + E[N_{NQ}]T_S + B + iV + (i-1)\rho L$ (21)

Combining the two cases above, the expected waiting time at the i^{th} node, $D_{i,j,NEQ}$, is then given by

$$D_{i,j,NEQ} = E[X_{i,j}] + L = E[X_{i,j,C1}]P[C1] + E[X_{i,j,C2}]P[C2] + L$$
$$= \frac{T_S}{2} + E[N_{NQ}]T_S + E[X_{FR}] - B - iV - jL$$

Unconditioning the above equation on j and recalling that j follows the Binomial distribution of Eqn. (3), the expected delay at the i^{th} node, $D_{i,NEQ}$ is given by

$$D_{i,NEQ} = \sum_{j=0}^{i-1} D_{i,j,NEQ} \binom{i-1}{j} \rho^j (1-\rho)^{i-j-1} = \frac{T_S}{2} + E[N_{NQ}]T_S$$
(22)

3.3 Overall Delay

The expressions for the delays of the previous two sections can now be combined to obtain the expression for the delay experienced by an arbitrary arrival. The expected packet delay at node i is given by

$$D_{i} = D_{i,EQ}P[EQ] + D_{i,NEQ}P[NEQ]$$

$$= \frac{T_{S}}{2} + \rho E[N_{NQ}]T_{S} + \left[\frac{\rho L^{2}(i-1)(1-\rho)}{T_{S}} + L\right](1-\rho)$$
(23)

where P[EQ], P[NEQ], $D_{i,EQ}$ and $D_{i,NEQ}$ are given in Eqns. (1), (2), (15) and (22) respectively. Note however, that the expression $E[N_{NQ}]$ is the expected number of packets seen an arrival given that the queue is non-empty. The expected number in the queue seen by an arbitrary arrival, $E[N] = \sum_{i=1}^{\infty} iP[N=i]$ is related to $E[N_{NQ}]$ by

$$E[N_{NQ}] = \sum_{i=0}^{\infty} \frac{iP[N=i, \text{NEQ}]}{P[\text{NEQ}]} = \sum_{i=1}^{\infty} \frac{iP[N=i]}{\rho} = \frac{E[N]}{\rho}$$
(24)

where P[N = i, NEQ] represents the joint probability that there are *i* packets in the queue and the queue is non-empty. Also from Little's Law

$$E[N] = \lambda D_i \tag{25}$$

Thus we have $E[N_{NQ}] = \lambda D_i / \rho$ and substituting this in Eqn. (24) we have the final expression for D_i

$$D_{i} = \frac{1}{1 - \lambda T_{S}} \left[\frac{T_{S}}{2} + \left(\frac{\rho L^{2}(i-1)(1-\rho)}{T_{S}} + L \right) (1-\rho) \right]$$
(26)

4 Simulation Results

In this section we validate the analytic models proposed in the previous sections by comparing them with simulation results. These simulations were carried out using the network simulator *ns* as well as our own simulation code. The simulations were carried out for different network sizes and parameter settings as indicated in in Table 4. The simulations considered a circular region of radius 240 meters with the base station at the center and all other nodes within its range.

In Figure 4 we compare the simulation and analytic results (from Eqn. (26)) for the unidirectional traffic case when there are 8 nodes in the network. We show two cases corresponding to CFPri values of 23 msec and 28 msec and in both cases we note the close match between the simulation and analytic results. The delays are presented for the 5^{th} node in the polling list. We also note that having a shorter CFPri supports higher arrival rates for a given delay requirement. Similar results were obtained for other network sizes, CFPri lengths and packet sizes and are not shown here due to space limitations.

Parameter	Value
Transmission power	$281.8~\mathrm{mW}$
Transmission range	250 meters
Slot time	$20 \ \mu sec$
SIFS	$10 \ \mu sec$
DIFS	$50 \ \mu sec$
PIFS	$30 \ \mu sec$
CFPriMax	30 msec
Channel bandwidth	2 Mbps
Beacon	209 μsec
CF-Poll	$209 \ \mu sec$
CF-End	$209 \ \mu sec$
CF-ACK	153 μsec
Packet size	520 B

 Table 1. Simulation Settings



Fig. 4. Simulation and analytic results for unidirectional transfer with 10 nodes.

5 Conclusions

In this paper, we presented an analytic model to evaluate the delays is wireless networks using the IEEE 802.11 PCF as the MAC layer protocol. Closed form expressions for the delays at each node were obtained using a queueing model for the cases with both unidirectional as well as bidirectional traffic. The model is able to account for arbitrary packet sizes, polling frequencies, channel rates and the order in which a node is polled. The accuracy of these expressions was verified using extensive simulations.

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