Coverage for Target Localization in Wireless Sensor Networks

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ABSTRACT

Target tracking and localization are important applications in wireless sensor networks. Although the coverage problem for target detection has been intensively studied, few consider the coverage problem from the perspective of target localization. In this paper, we propose two methods to estimate the necessary sensor density which can guarantee a localization error bound over the sensing field. In the first method, we convert the coverage problem for localization to a conventional disk coverage problem, where the sensing area is a disk centered around the sensor. Our results show that the disk coverage model requires 4 times more sensors for *tracking* compared to *detection* applications. We then introduce the idea of sector coverage, which can satisfy the same coverage conditions with 2 times less sensors over the disk coverage approach. This shows that conventional disk coverage model is insufficient for tracking applications, since it overestimates the sensor density by two times. Simulation results show that the network density requirements derived through sector coverage are close to the actual need for target tracking applications.

Categories and Subject Descriptors: C.2.1 [Network Architecture and Design]: Wireless communication

General Terms: Design, Performance.

Keywords: Sensor networks, Coverage, Target Tracking.

1. INTRODUCTION

One of the fundamental tasks for Wireless Sensor Networks (WSNs) is to collect information from the physical world. A network designer faces several challenges when designing a wireless sensor network. Apart from the wireless medium, the primary challenges for wireless sensor networks stem from two facts. First, sensor nodes are extremely resource constrained. Second, in many applications sensor nodes will be randomly deployed. This randomness raises the issue of self-organization and equally importantly that of dimensioning the network. Scattering too few nodes may result in lack of coverage of the sensor field and a disconnected network. Scattering too many nodes may result in an inefficient network due to increased MAC collisions and interference. It is this critical aspect of dimensioning the network that we address in this paper.

The dimensioning problem from the point of view of *coverage* and *connectivity* has been intensely studied in recent years [1]. The most commonly used model in coverage problems is the disk model, which assumes that the sensing region for a sensor is a circular region centered around it. A point in the field is said to be covered if it is within the sensing region of at least one sensor. The disk model has certain limitations in describing the information loss. First, it does not consider the cooperative detection of multiple sensors. When several nearby sensors are monitoring an event at the same time, the estimation error can be reduced through cooperative signal processing [2]. In this case, although each single sensor may not be able to provide precise information about the event, the information loss can still be tolerable when we combine the measurements from multiple sensors. Thus, the sensing region for a cluster of sensors can be much greater than the union of their coverage disk. Second, the disk model is also inadequate in describing the information loss of certain applications. For example, in target tracking application, the network objective is to localize the target within a certain error margin. Ensuring that the region is covered by sensing disks can not provide guarantees for precise localization. Even with the concept of *k-coverage*, where every point should be in the sensing range of at least k sensors, the localization error still can not be tightly bounded. Consider the case where a point is covered by k sensors clustered together. Although the point is k covered, the localization error may still be large since the k sensors only provide redundant location information about the target.

In this paper, we investigate the coverage problem for an important sensor network application – target tracking application. We first define the *coverage hole* for tracking applications as areas where the network can not localize the target within a predefined error bound. We then use the concept of network resolution on localization to bound the localization error over the network. Normally, a network with higher sensor density has better resolution. Given a requirement on localization error, we first convert the requirement to a bound on network resolution. Then we estimate the necessary sensor density based on the network resolution, since these two parameters are closely related.

We propose two methods to derive the relationship between sensor density and network resolution. The first method

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is based on a sufficient condition which can guarantee certain resolution over the whole field. Using this sufficient condition, the coverage for localization can be converted to a conventional disk coverage problem. Under the disk model, we show that we need 4.64 times more sensors for tracking applications compared to detection. However, the actual sensor density required for localization is much lower than that estimated by the disk model. In order to estimate the necessary density more precisely, we introduce the idea of sector coverage. By using the sector coverage model, we get a localization error bound with only half the density required by disk coverage. Comparing the two methods, we see that the disk coverage model is less accurate, since the density bound provided by this model is quite loose. However, the disk model can utilize the existing coverage algorithms, while the sector coverage model is much more complex. Our experimental results are close to the analytical results of the sector coverage, proving the usefulness of our analytical methodology.

The rest of this paper is organized as follows: Section 2 summarizes some previous work on coverage and localization problems. The system model and the definition of coverage in tracking applications are described in Section 3. Section 4 gives the sufficient condition to provide bounded localization error and further relates this condition with disk coverage model. Section 5 gives a better density estimation method based on the idea of sector coverage. The experiment results are provided in Section 6. Finally, Section 7 concludes the whole paper.

2. RELATED WORK

The coverage problem in wireless sensor networks has been intensively studied in recent years [3]. Most of these works look at the problem of covering every point in the sensing field with sensing disks [4, 5] or detecting a target when it passes through the sensing field [6, 7]. In this paper, we investigate the coverage problem from a different view point by addressing the localization error of the sensor network. However, the results and algorithms in the previous works can also apply to the disk coverage conditions derived in Section 4.

Localization is an important part in sensor networks, since it provides coordinates both for sensors [8] and for the targets in the sensor network [9]. In this paper, we focus on the problem where the sensor locations are already known and the objective is to localize targets in the sensing field. Several well known estimation methods, such as Cramér-Rao Bound (CRB) [10] or Bayesian Bound (BB) [11], are used in range based localization systems. In this paper, we use a simplified localization model for the coverage problem. We try to bound the location estimation error to be within a small circle of radius ε with high probability. This is different form normal approach to minimize the Mean Square Value of the localization error. Our model is reasonable for systems where the performance is determined by the maximal localization error.

Connecting the coverage problem with the localization problem will provide useful guidelines for sensor network deployment [12]. Beacon deployment for sensor localization is studied in [13, 14]. Nagpal et al. point out that there should be voids around areas where the localization error is large [15]. Their results show that for minimal neighborhood size larger than 15, the localization error is small enough compared to the communication range. However, their localization system is based on hop counts and focus on the localization for sensors. In this paper, we provide localization error bounds for range based systems and our results can fit in wider scopes.

3. SYSTEM MODEL

We assume that the sensor network is running a target tracking application, where the objective of the network is to provide accurate location information of the target. We assume that the coordinates of sensors are known, and the location of the target is estimated based on the measurements and coordinates of nearby sensors. We focus on networks formed by sensors which can measure their distance to the target, e.g., Time Difference Of Arrival (TDOA) and Received Signal Strength Indicator (RSSI) sensors.

Due to the existence of noise, the distance estimation will be distributed within a certain range around the true distance. When the true distance between sensor S_i and the target is $d_{i,t}$, we assume the estimated distance $\tilde{d}_{i,t}$ by sensor i will fall in the range $[d_{i,t} - e_l, d_{i,t} + e_u]$ with high probability, where e_l and e_u are the error bounds. We set e_l, e_u equal to e in the latter derivation. However, our result can also be easily extended to the case where e_l and e_u are not equal. We assume that sensors can only provide distance measurements when the target is within its detection range of r. We define the *normalized measurement accuracy* of sensor as $K = r/e$, which is the ratio of the detection range divided by the maximum measurement error within the detection range. Under the assumptions we make, when sensor S_i gives a distance estimate of $\tilde{d}_i$, the target will fall in $[\tilde{d}_{i,t}-e, \tilde{d}_{i,t}+e]$ with high probability. Then, a single sensor i can localize the target within an annulus of $[\tilde{d}_{i,t}-e, \tilde{d}_{i,t}+e]$, as shown in Fig. 1.

When the target can be detected by several sensors at the same time, the position estimation can be further refined by cooperative signal processing. In this paper, we combine the sensor measurements by intersecting the annuli of different sensors [16], as shown in Fig. 1. The intersection area is defined as the high probability *uncertainty region*. As more sensors provide distance information about the target, the uncertainty region will become smaller. Given the group of distance measurement $\tilde{d}_{1,t}, \tilde{d}_{2,t}, \ldots, \tilde{d}_{k,t}$ provided by all the k sensors which can detect the target, we can determine the smallest uncertainty region. The estimation of the target location can be set as the center of the smallest circle which can circumscribe this smallest uncertainty region. The localization error is smaller than the radius of this circle, since the distance from all points in the uncertainty region to the center is smaller than the radius. If the sensor density is high enough, we can make all uncertainty regions small enough to be well contained in circles with radius smaller than ε. In this case, the localization error is always smaller than ε, no matter where the target is. Note that our localization method is much simpler compared to currently used localization algorithms. However, this simplified model retains the basic ideas of range based localization while at the same time revealing key insights and relationships between the coverage and localization problem.

Now we define the concept of coverage under this localization model. For the tracking application, we require the location estimation error to be within a circle of radius ε. In a randomly deployed sensor network, there may exist areas

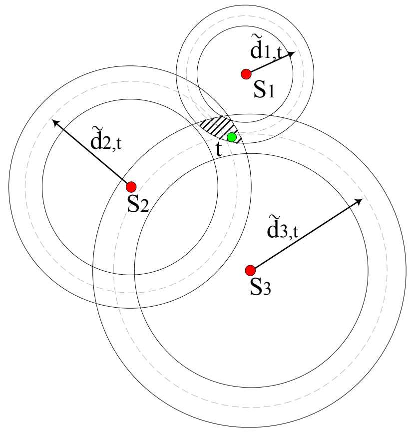

Figure 1: Each of the three sensors S_1,S_2 and S_3 can provide distance information of the target t.The target location will be within the intersection of the annuli around them, shown as the shaded area.

where local sensor density is so low that we can not precisely localize the target in these areas. The *coverage hole* is defined as holes where the localization error exceeds the predefined bound. The objective of this paper is to find the sensor density which can ensure that there is no coverage hole in the network or the area of coverage hole is small compared to the total area of the field.

4. COVERAGE IN TRACKING APPLICATIONS

4.1 Sufficient Condition for Coverage

In this section, we give the sufficient condition for the localization error to be bounded for all positions in the field. We first introduce the concept of network resolution on localization. We say that two point t and t' are distinguishable, if the sensor network can always distinguish whether a target is at point t or at t' through the distance measurements provided by sensors. The *network resolution* is the minimum distance l, such that the network can distinguish any pair of points when the distance between them is larger than l. The network resolution is related to the localization error bound ε by the following lemma.

LEMMA 1. *If the network resolution is better than $\sqrt{3}\varepsilon$, then the location estimation error is upper bounded by ε. If the network resolution is worse than 2ε, then the location estimation error is lower bounded by ε.*

PROOF. We see that points in the same uncertainty region can not be distinguished by the network. If the network resolution is better than $\sqrt{3}\varepsilon$, then every uncertainty region should not contain two points apart by more than $\sqrt{3}\varepsilon$. Thus, the *Generalized Diameter*, which is defined as the greatest distance between any two points in the shape, is smaller than $\sqrt{3}\varepsilon$ for all uncertainty regions. A hexagon with side length of ε can fully cover any shape with Generalized Diameter smaller than $\sqrt{3}\varepsilon$ [17]. Thus, the circumcircle of such a hexagon, which has a radius of ε, can also cover all uncertainty regions. Using the center of the covering circle as the estimated location of the target will provide estimation error smaller than ε. This bound is tight since when uncertainty region is shaped as an equilateral triangle with side length of $\sqrt{3}\varepsilon$, the smallest circle which can circumscribe it has radius of exactly ε.

If the network resolution is worse than 2ε, then there should be two points in the same uncertainty region that are apart by more than 2ε. Such an uncertainty region can not be circumscribed by circles with radius smaller than ε. Thus, the estimation error is larger than ε. □

THEOREM 1. *If there is at least one sensor in any arbitrarily selected sector of radius r (the predefined detection range) and angle $\frac{2\pi}{3}$, denoted as a sector of $(r, \frac{2\pi}{3})$, then the location estimation error is bounded by $\varepsilon = \frac{4\sqrt{3}e}{3}$ over the network, where e is the maximal distance estimation error when target is within the detection range r.*

PROOF. As shown in Lemma 1, if the network resolution is better than $\sqrt{3}\varepsilon = 4e$, we can guarantee a location estimation error bound of ε. We will prove this by contradiction. Suppose the network resolution is worse than $4e$, which means we can find at least one pair of points t and t' which are apart by more than $4e$, but no sensor can distinguish them.

Suppose the true target position is t, and sensor S_i can detect it. According to the assumption of the distance measurement error, the distance measurement $\tilde{d}_{i,t}$ provided by sensor S_i should satisfy:

$$d_{i,t} - e \le \tilde{d}_{i,t} \le d_{i,t} + e \quad (1)$$

where $d_{i,t}$ is the Euclidian distance between the sensor i and the target t. Eq. (1) can also be rewritten as:

$$\tilde{d}_{i,t} - e \le d_{i,t} \le \tilde{d}_{i,t} + e \quad (2)$$

which shows the true distance of $d_{i,t}$ must be within $[\tilde{d}_{i,t} - e, \tilde{d}_{i,t} + e]$.

Suppose sensor S_i can not distinguish point t from another point t', i.e., given a measurement of $\tilde{d}_{i,t}$ the target can either be at point t or t'. Then the measurement of $\tilde{d}_{i,t}$ should also be a valid value when the target is at point t'. Thus, the true distance of $d_{i,t'}$ must be within $[\tilde{d}_{i,t} - e, \tilde{d}_{i,t} + e]$ too. Otherwise S_i will not give distance estimation of $\tilde{d}_{i,t}$ when target is at t'.

Since both $d_{i,t}$ and $d_{i,t'}$ is in the range of $[\tilde{d}_{i,t} - e, \tilde{d}_{i,t} + e]$ for some $\tilde{d}_{i,t}$, the difference between $d_{i,t}$ and $d_{i,t'}$ should satisfy:

$$-2e \le d_{i,t} - d_{i,t'} \le 2e \quad (3)$$

Eq. (3) shows that if there is a sensor S_i whose distance to points t and t' differs by more than $2e$, then sensor S_i can definitely distinguish the point t from point t', given that it can detect at least one of these two points. So, if there are two indistinguishable point t and t', all sensors should either be outside the detection range of these two points or within the areas where the distance to t and t' are nearly equal.

Suppose point t and t' can not be distinguished by the network, and the distance between t and t' is $4e$. As we assumed, there is at least one sensor in every arbitrarily selected sector of $(r, \frac{2\pi}{3})$. So, there must be at least one sensor within range r of point t. Suppose this sensor is sensor S_i.

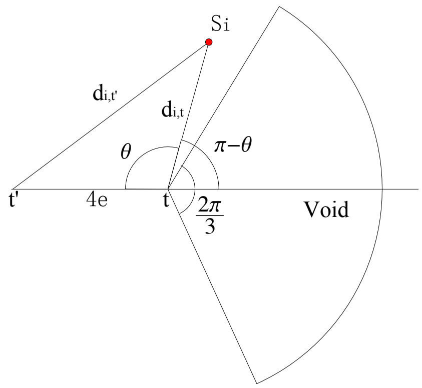

Figure 2: Sensor S_i can detect both point t and t', but can not distinguish them.

Then sensor S_i should be able to detect the point t', otherwise these two points can not be in the same uncertainty region and the network can distinguish them. Without loss of generality, suppose sensor S_i is closer to the point t, i.e., $d_{i,t} < d_{i,t'}$, as shown in Fig. 2. From Eq. (3), we have $d_{i,t'} \leq d_{i,t} + 2e$, then:

$$\begin{aligned} \cos\theta &= \frac{d_{i,t}^2 + (4e)^2 - d_{i,t'}^2}{2 \times d_{i,t} \times 4e} \\ &\geq \frac{12e^2 - 4d_{i,t}e}{8d_{i,t}e} \\ &\geq -\frac{1}{2} \end{aligned} \tag{4}$$

Since $\cos\theta$ is monotonically decreasing in $[0, \pi]$, the angle θ must be smaller than $\arccos(-\frac{1}{2}) = \frac{2\pi}{3}$, so the angle $\pi - \theta$ is larger than $\frac{\pi}{3}$. Therefore, no sensor can be within the sector of $(r, \frac{2\pi}{3})$ centered at t and bisected by ray $t't$. This is a contradiction to the assumption that there should be at least one sensor in any arbitrarily selected sector of $(r, \frac{2\pi}{3})$.

Eq. (4) considers the case that $d_{t,t'} = 4e$. If the distance between t and t' is larger than $4e$, this lower bound for $\pi - \theta$ increase monotonically. Thus, if the two indistinguishable points t and t' are separated by more than $4e$, there also should be no sensors in the sector of $(r, \frac{2\pi}{3})$ centered at t, which contradicts to our assumption.

Therefore, there does not exist indistinguishable points t and t', which are apart more than $4e$. Thus, the network resolution is better than $4e$. This directly lead to the localization error bound of $\frac{4\sqrt{3}e}{3}$ by lemma 1. □

Theorem 1 can be directly extended to the disk coverage model as follows.

Corollary 1. *Given a sensor deployment, if disks of radius $\frac{\sqrt{3}}{\sqrt{3}+2}r$ centered at the sensors can cover the entire field, then the location estimation error is bounded by $\varepsilon = \frac{4\sqrt{3}e}{3}$ over the network.*

Corollary 1 comes from the fact that a sector of $(r, \frac{2\pi}{3})$ contains a disk of radius $\frac{\sqrt{3}}{\sqrt{3}+2}r \approx 0.464r$, as shown in Fig 3[1]. If there exists a $(r, \frac{2\pi}{3})$ sector void, then from Theorem 1, there is no sensor within a radius of $0.464r$ from the center point A. Therefore, if the sensor node density is high enough that the field is totally covered by disks of radius $0.464r$ centered at the nodes, then there will be no $(r, \frac{2\pi}{3})$ sector void and the location estimation error bound is guaranteed. Thus, we can directly use the known results in disk coverage for target tracking application by shrinking the radius of coverage disk.

Corollary 1 shows that we need to deploy sensors at a higher density when we need to localize the target than simply detect the target. We can estimate the density required for localization coverage as follows. Assume we have to provide coverage over a square field of area A. We can estimate the density required by translating this problem to the following equivalent disk coverage problem. If we look at a square field with sides scaled by a factor of 0.464, the number of sensors required to cover this field with sensing radius $0.464r$ is the same as the sensors required to cover the area of A with radius r. However, the same number of sensors only covers $0.464^2 A$ area with a shrunk sensing radius of $0.464r$. Therefore the node density required for localization coverage is $\frac{1}{0.464^2} \approx 4.64$ times more than that required for a detection coverage.

Shrinking the sensing range to $0.464r$ is actually necessary in the disk coverage model. Consider the case where sensors are deployed densely on the border of the shaded area in Fig. 3, if we check the coverage by using sensing disks larger than $0.464r$, the algorithm will consider the whole area is covered (or even k covered). However, in this case, none of these sensors can distinguish point t from t' and the localization error can exceed the bound.

[1]Note that Eq. (4) only gives a lower bound of the area. The exact white area drawn here, which is derived by Eq. (3), is larger than a $(r, \frac{2\pi}{3})$ sector.

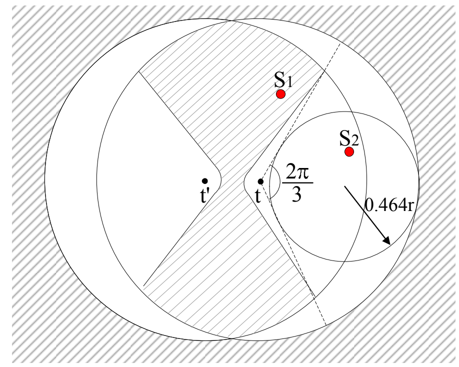

Figure 3: Sensor in the shaded area, such as S_1, can not distinguish the point t form t'. Sensors in the white area, such as S_2, can distinguish these two points, so there should not be any sensors in the white area.

4.2 Relationship between Resolution and Density

Theorem 1 shows how to guarantee network resolution of $4e$. As the uncertainty region of single sensor is an annulus with width of $2e$, we may further improve the network resolution to $2e$ by increasing the node density. However, we show below that this will not be cost efficient.

Suppose that we want to guarantee network resolution[2] of αe with $\alpha \geq 2$. Substitute this into Eq. (4), we have:

$$\begin{aligned} \cos\theta &= \frac{d_{i,t}^2 + (\alpha e)^2 - d_{i,t'}^2}{2 \times d_{i,t} \times \alpha e} \\ &\geq \frac{(\alpha e)^2 - 4 d_{i,t} e - 4e^2}{2\alpha d_{i,t} e} \\ &\geq -\frac{2}{\alpha} \end{aligned} \quad (5)$$

When $d_{i,t}/e$ is large, θ_{max} can be exactly equal to $\arccos(-\frac{2}{\alpha})$. As α decrease from 4 to 2, θ_{max} grows from $2\pi/3$ to π. As shown in the proof of Theorem 1, we require every sector of $(r, 2(\pi - \theta))$ to contain at least one sensor. For small α value, the angle of such sectors decreases to zero, thus it requires extremely high density to make network resolution close to $2e$.

To demonstrate this, we investigate the relationship between α and sensor density under the disk coverage model. The maximal radius for a circle which can be packed in the $2(\pi - \theta)$ sector is $\sin\theta/(1 + \sin\theta)$. The relationship between the density and network resolution is plotted on Fig. 4. When the resolution requirement is close to $2e$, the density increases quickly, yet the gain in network resolution is small. On the other hand, when the network resolution is worse than $4e$, the density nearly remain constant and it converges to 4 times the detection density as α goes to infinity. The radius of the sensing disk is upper bounded by $0.5r$ when we increase α. This means that we need at least one sensor in disks of radius $0.5r$ to meet the basic density requirement for localization, e.g., the node is detected by at least 3 sensors at appropriate positions for triangulation.

Fig. 4 hints that the network resolution of $3e$ to $4e$ is the best trade-off between the node density and the network resolution. Therefore, when sensors with low accuracy are used, we can not simply increase the node density to achieve high network resolution. Normally the distance measurements error decreases as the sensors are close to the target. A better way to increase the network resolution is to decrease e by shrinking the detection range r, i.e., only use sensors which can provide more accurate distance information in the localization.

4.3 The density gain of cooperation

Our localization model requires all sensors which can detect the target to exchange distance information with each other and do cooperative signal processing for localization. This localization process may cause more communication and computation overheads than non-cooperative approaches. However, such overheads are worthwhile since the cooperation can greatly reduce the network density.

Consider a localization model with less cooperation: We use the position of the sensor which is closet to the target as the position of the target. Suppose sensors broadcast the location information of the target in a contention based manner, the closest sensor have a better chance to send first. In this case, the location information may only be send out once, since other sensors will be suppressed by the sensor closest to the target. Comparing to the cooperative approach, which requires at all sensors in the detection range to exchange their estimations, the communication cost can

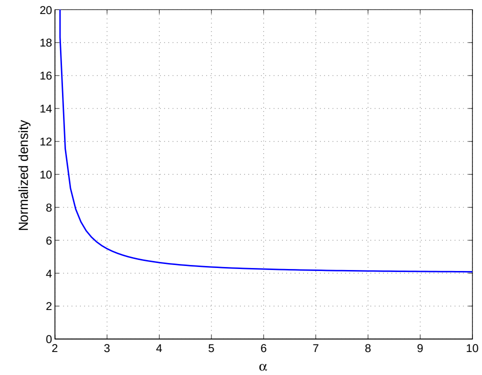

Figure 4: The relationship between the network resolution and the network density. α is network resolution divided by e, the distance measurement error. The normalized density is ratio of localization coverage compare to detection coverage.

be reduced by a factor which is proportional to the network density. However, it is difficult to reduce the localization error in the non-cooperative approach. The localization area for this model is same as the *Voronoi Cells* around the sensor, which is defined as the set of all points that are closer to this sensor than any other sensors[3]. To make sure that the target is within a circle of radius ε, all Voronoi Cells must be smaller than a circle of radius ε centered at each sensor, which means the sensor coverage range is smaller than ε. Comparing to the coverage range of $0.464r$ for the cooperative scheme, the coverage range for non-cooperative scheme is $\varepsilon = \frac{4\sqrt{3}r}{3K}$, which is about $\frac{4\sqrt{3}r}{3K}/0.464r \approx 5/K$ times of the cooperative case. So, the non-cooperative scheme needs $(0.2K)^2$ more sensors to provide the same network resolution. Considering most sensors have accuracy $K = r/e$ much larger than 100 [10], cooperation actually makes the density for the network hundreds times lower. Such advantage makes the overheads for cooperation justifiable, and almost all localization systems which can provide distance measurements use cooperative schemes.

5. DENSITY ESTIMATION THROUGH SECTOR COVERAGE

From the proof of theorem 1, we can see that when the localization error exceeds the bound, there are voids shaped as white area as in Fig. 3. Showing that there is no $0.464r$ radius disk void in the network can eliminate the existence of such sector shaped void. However, a network may not contain such sector shaped void even if it is not covered by $0.464r$ disks. Thus, the sufficient condition derived by disk coverage is too strict. It may give a higher estimation on sensor density than actually required.

In this section, we introduce the concept of *sector coverage* to provide better estimation on the necessary density for a randomly deployed network to guarantee bounded localization error. The derivation is based on the assumption that sensors are randomly scattered in the field and the sensor distribution follows a stationary Poisson point process with

[2]Since the annulus used in localization has width of $2e$, it is impossible to guarantee network resolution smaller than $2e$.

[3]We assume the distance estimation error is small in this case, since the sensors are very close to the target.

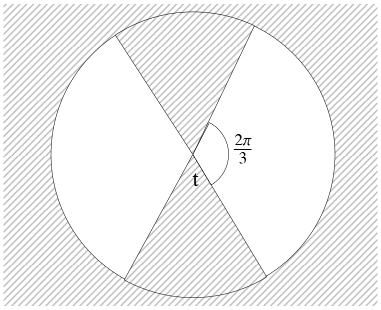

Figure 5: Approximating the white area in Fig.3 with two sector areas. If there exist any void as the white area (two opposing sector with angel of $\frac{2\pi}{3}$) around t, then t is not covered.

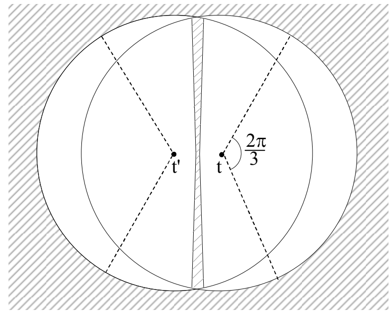

Figure 6: When t and t' is far apart, the shaded area in Fig.3 shrink to narrow strip.

intensity λ [18]. We use average vacancy over unit area to measure the quality of coverage over a field [18]. The average vacancy over unit area is defined as ratio of uncovered area divided by the total area of the field. It is equal to the probability that an arbitrary point is not covered, i.e., the localization error exceeds the bound at that point.

Instead of estimating whether there are voids shaped as the white area in Fig. 3, we approximate the voids by two opposing sectors of $(r, \frac{2\pi}{3})$ around point t, as shown in Fig. 5. We call this approximation as *sector coverage*: a point is sector covered if there is at least one sensor in any pair of opposing sectors of $(r, \frac{2\pi}{3})$ around it.

The approximation of sector coverage is based on the following observations. Suppose that t is uncovered, then there will be a point t' which can not be distinguished from point t and $d_{t,t'}$ is larger than $4e$. Using the arguments in Theorem 1, we see that there should also be a sector void of $(r, \frac{2\pi}{3})$ around t' by symmetry. When the sensor accuracy K is high, the two white areas around t and t' can be treated as two opposing sectors of $(r, \frac{2\pi}{3})$ around point t. So, if there is no such opposing sector void in any orientation around t, the chance that there exists an indistinguishable point t' around t is small. We validate this approximation in two cases.

First, consider the case that $d_{t,t'}$ is comparable to $4e$. Since we assumed that K is large, so $r \gg e$. As the distance between t and t' is small comparing to r, the two circles in Fig. 3 will almost completely overlap with each other. Also, the angle θ defined in Eq. (4) converges to $\frac{2\pi}{3}$ as $d_{i,t}$ approaches r, which means the border of the shaded area in Fig. 3 will overlap with the border of the sector. Thus, the difference between $\frac{2\pi}{3}$ sector and the white area in Fig. 3 can be ignored in this case.

Second, consider the case that $d_{t,t'}$ is much larger than $4e$. To satisfy Eq. (3), the shaded area shrinks to a narrow strip as shown in Fig. 6. Since the width of this strip is of the order of e, the area of this strip is much smaller than the total area of the circle. Thus, the chance that two or more sensors fall in this area is small and can be ignored. When there is only one sensor falling in the strip, there is only one sensor in the detection range of point t. In that case, there should definitely be two opposing sector voids around t, and this situation is already included in the sector coverage test. Section 6 will further validate our approximation by simulations.

We now proceed to find the density requirement for sector coverage. Consider a Poisson point process with intensity λ, the probability that there are k sensors in the detection range r of point t is given by:

$$P_k = e^{-\pi\lambda r^2} \frac{(\pi\lambda r^2)^k}{k!} \tag{6}$$

For sensors falling within the range r, we denote the sensor's position as (a_i, ϕ_i) in polar coordinates with the origin at t. Using the property of Poisson process, ϕ_i are independent and uniformly distributed over $[0, 2\pi]$ given there are k sensors in the circle.

If there are less than one sensor in the detection range, we can always find two opposing $(r, \frac{2\pi}{3})$ sector voids, thus, the point t can not be sector covered in this case.

If there are two sensors within range r and the angle between these two sensors falls in the range of $[-\frac{\pi}{3}, \frac{\pi}{3}] \cup [\frac{2\pi}{3}, \frac{4\pi}{3}]$, then the point t is not sector covered[4]. Since the angle $\phi_1 - \phi_2$ is uniformly distributed too, the probability that a point t is not sector covered given there are two sensors in the detection range is $\frac{2}{3}$.

If there are $k > 2$ sensors in the circle, we first convert the problem of detecting two opposing sector voids of $\frac{2\pi}{3}$ to detecting one continuous sector void of $\frac{4\pi}{3}$. For every sensor, define ϕ'_i as

$$\phi'_i = \begin{cases} 2\phi_i, & 0 \le \phi_i < \pi, \quad i = 1, 2 \ldots, k \\ 2(\phi_i - \pi), & \pi \le \phi_i \le 2\pi, \quad i = 1, 2, \ldots k \end{cases} \tag{7}$$

Two opposing sensors with angle of ϕ_i and $\phi_i + \pi$ will transformed to the same angle of $\phi'_i = 2\phi_i$. Thus, any sensor deployment that has void of two opposing sector of $\frac{2\pi}{3}$ will have a void of $\frac{4\pi}{3}$ in the transformed coordinate system of (a_i, ϕ'_i). It is easy to see that the angle ϕ'_i is also independent and uniformly distributed on $[0, 2\pi]$.

For k independently uniformly distributed ϕ'_i, the probability that the range of the samples, defined as $\max\{\phi'_i\} - \min\{\phi'_i\}$, is smaller than $\frac{2\pi}{3}$ is given by [19]:

$$Q_k = k\left(\frac{1}{3}\right)^{k-1} - (k-1)\left(\frac{1}{3}\right)^k \tag{8}$$

[4]Note that sector coverage is an approximation. A point can be sector covered when there are only two sensors in the detection range, yet the localization error may exceed the bound.

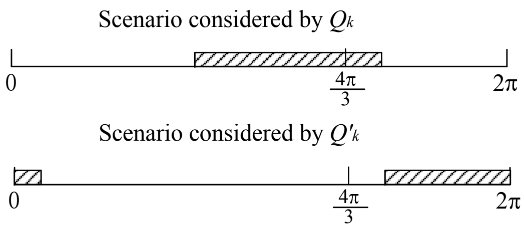

Figure 7: Bounding the probability that there exists a sector void of $(r, \frac{2\pi}{3})$, the shaded area means all sensors are confined in that angle.

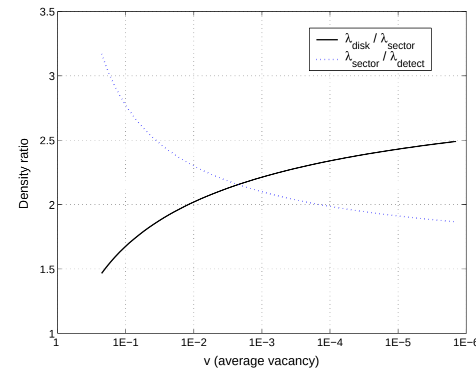

Figure 8: Comparing the density requirements for disk coverage and sector coverage.

Q_k is the probability that all k sensors are confined in a $\frac{2\pi}{3}$ sector with the remaining $\frac{4\pi}{3}$ sector as a void. Since sectors can cross the zero angle, Q_k does not account for the case that all k sensors are confined in a sector with an angle smaller than $\frac{2\pi}{3}$ and it crosses the zero angle, see Fig. 7. We define the probability of this scenario as Q'_k. It is clear that Q'_k is smaller than the probability that k sensors are confined in a $(r, \frac{2\pi}{3})$ sector *and* the sector starts from $[\frac{4\pi}{3}, 2\pi]$. This probability is half of the probability that sensors are confined in a $(r, \frac{2\pi}{3})$ sector *and* the sector starts from $[0, \frac{4\pi}{3}]$ due to the uniform distribution. Since Q_k includes all cases that sensors are confined in a $(r, \frac{2\pi}{3})$ sector *and* the sector starts from $[0, \frac{4\pi}{3}]$, Q'_k is upper bounded by $Q_k/2$.

Combining all these cases, the sector coverage probability for an arbitrary point t is bounded as:

$$P_0 + P_1 + \frac{2P_2}{3} + \sum_{k=3}^{\infty} Q_k P_k \le P_{sector}$$
$$\le P_0 + P_1 + \frac{2P_2}{3} + \frac{3}{2}\sum_{k=3}^{\infty} Q_k P_k \qquad (9)$$

Actually these two values are very close to each other. In most cases, we need to upper bound the probability of vacancy, so we only use the upper bound in what follows.

For disk coverage, the probability that one point is not covered is given by:

$$P_{disk} = e^{-\pi\lambda(0.464r)^2} \qquad (10)$$

which is the chance that there are no sensors in the disk of radius $0.464r$. For a given coverage requirement of average vacancy v over a unit area, which is the same as the probability for an arbitrary point to be uncovered, we can calculate the necessary density for both sector coverage and disk coverage, denoted as λ_{disk} and λ_{sector}.

Fig. 8 shows that the density requirement for sector coverage is much smaller than that obtained by translating the localization coverage problem into an equivalent disk coverage problem. As v becomes smaller, $\lambda_{disk}/\lambda_{sector}$ converges to 2.5, which means sector coverage requires 2.5 times fewer sensors than that obtained from equivalent disk coverage method. This indicates that the concept of disk coverage is not suitable for describing the density requirement for localization. For the sector coverage, the density requirement for localization is nearly 1.8 times higher than the density required for detection coverage when v is small.

6. EXPERIMENT RESULTS

6.1 Experiment Settings

In this section, we use Monte Carlo methods to verify our theoretical results. The experiment setting is as follows. We randomly deploy λA sensors on a large region with area A, so we approximately get a Poisson point process with intensity λ in a small area inside A. We check whether there exist a pair of points which can not be distinguished by the randomly deployed sensors. The minimum distance between two points which can not be distinguished is the network resolution. We repeat over 10,000 iterations to obtain the probability that network resolution is worse than αe. In this way, we get the relationship between the network resolution and the sensor density.

6.2 Average Vacancy

The average vacancy is the probability that the network resolution is worse than the predefined bound of $4e$. The experiment results are shown in Fig. 9. The sector coverage provides good estimations for the average vacancy when the sensor accuracy K, is larger than 50. For small K values, since the white area in Fig. 3 is much larger compared to the $(r, \frac{2\pi}{3})$ sector approximation we used in the sector coverage, it is much easier to get a point to be covered. Thus, the average vacancy is smaller than the estimated value. The vacancy estimation by the equivalent disk coverage method is much larger than the experimental result, which means the disk model is not accurate in estimating the vacancy. Sector coverage provides better estimates, however it is more complex for sensors to determine whether the area around it is sector covered in this model.

6.3 Network Resolution

Recall that we can represent the network resolution as αe, where e is the distance estimation error. Fig. 10 shows the average vacancy under different α values. For $\alpha = 2$, more than 95% network area remain as vacancies even the network density are high. This verifies our theoretical results in Section 4.2, which shows it is difficult to get localization error of $2e$ by increasing sensor density. For $\alpha > 3$, the average vacancy decreases sharply as the node density increases. For a given network density, the average vacancy decreases as α increase. This is because higher tolerance for

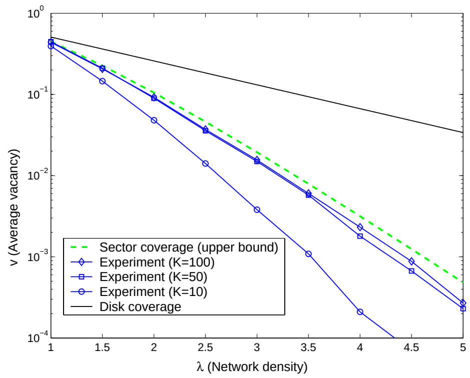

Figure 9: The experiment results on average vacancy for different sensor accuracy.

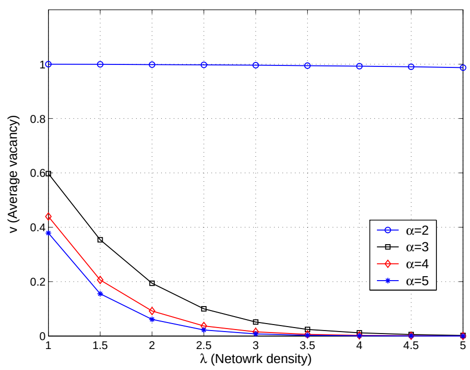

Figure 10: The experiment result on average vacancy for different network resolution ($K = 100$).

network resolution is reflected in larger α values. However, the difference become smaller when α is large. The curve for $\alpha = 4$ is very close to the one for $\alpha = 5$, which means achieving $\alpha = 4$ requires almost the same density for $\alpha = 5$, so it is cost efficient to chose α value close to 4. This result coincides with our theoretical analysis in Section 4.2.

7. CONCLUSION

In this paper we have investigated the coverage problem for target tracking applications. We proposed two methods to illustrate the sensor density required for satisfactory localization in a field. In the first method we transform the problem into an equivalent disk coverage problem and show that we require 4 times more sensors in tracking applications than that in detection applications. However, if we use the method of sector coverage, the density required is only 1.8 times more than that for detection. Moreover, this method provides very tight bounds on the required density. This shows conventional disk coverage model is insufficient for tracking applications. Therefore, when we apply the disk model in coverage problems, we need to be aware of the trade-off between the simple implementation of the disk model and the potential overestimation of the sensor density due to using this simplified model.

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