On the Throughput Comparisons of MAC Protocols in Multi-hop Wireless Networks

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***Abstract*—We revisit the use of throughput metrics in studying Medium Access Control (MAC) protocols in static multi-hop wireless networks. To complement existing single-hop and multi-hop throughput notions, we first propose a unified normalized throughput expression. Since current multi-hop metrics do not give much intuition on how close a MAC protocol's throughput performance is to the best achievable for a given network topology, we present a new variant that benchmarks against the maximum achievable throughput. It can be characterized by the product of the number of maximum successful simultaneous transmissions $s_{\max}$ under saturated traffic conditions, and link rate R. We show how to compute $s_{\max}$ via a linear programming formulation and demonstrate its use in both string and grid topologies. We also derive exact mathematical expressions for the maximum simultaneous transmissions for these two topologies.**

***Index Terms*—MAC protocol evaluation, multi-hop throughput comparisons, throughput metrics, performance benchmarking.**

I. Introduction

By exploiting spatial reuse in multi-hop settings, nodes sufficiently far apart can transmit simultaneously to improve overall network performance. To this end, there are many works that focus on medium access control (MAC) protocol design and its evaluation [1]–[10]. Rather than proposing yet another protocol, we explore the use of throughput metrics in the performance evaluation of MAC protocols.

Throughput in single-hop networks is well understood [10]. The computed throughput in bits per second (bps) can be normalized with the link transmission rate R (assuming that all nodes use the same rate), so that the resultant value is within the range [0,1]. This is essentially benchmarking against the maximum achievable throughput since only a single node can successfully deliver its data packets at rate R, in a network with negligible propagation delay. Unfortunately, this is not so straightforward for multi-hop networks. From the literature, two commonly adopted throughput metrics for multi-hop networks are: *aggregate throughput* (also called network/system throughput) [1]–[3] and *throughput per-node* (also called per-station/per-user throughput) [4]–[6]. Aggregate throughput is the summation of the throughputs of all nodes in a network. For throughput per-node, the aggregate throughput is divided by the total number of nodes. Some works also normalize throughput per-node by link rate R [7]–[9]; this is called *rate-normalized throughput per-node*.

We now explain two limitations that arise from the use of the aforementioned multi-hop throughput metrics. First, these metrics are not a generalization of the single-hop throughput metric, in that they do not seek to benchmark against the maximum achievable throughput. Note that the maximum achievable throughput in multi-hop networks is different from the single-hop case. Here, the quantity we are interested in is the maximum *aggregate* data rate that can be supported for all nodes simultaneously in a given network topology, rather than only a single node's rate R. Hence, the existing multi-hop throughput metrics do not provide as much intuition as the single-hop throughput metric, with regard to the performance relative to the best achievable. For instance, the use of rate-normalized throughput per-node often results in a very low normalized value (e.g., on the order of 10^{-2} in [7]–[9]), and does not provide any hint about how far it is from the best achievable. As another example, we will show later that when Aloha is applied in a multi-hop network with string topology, the peak throughput is actually quite close to the theoretical peak for single-hop networks, which is around 18% of the maximum achievable throughput. This cannot be appreciated if the rate-normalized throughput per-node metric were used, as it only gives a normalized, unitless value of around 0.09.

Second, many previous works only compare a proposed MAC protocol's throughput against that of a de facto MAC, such as the IEEE 802.11 [1]–[3]. This comparison approach is inadequate for performance analysis, because it only portrays a *relative* performance improvement/degradation. Instead, a better approach would be benchmarking with respect to the best achievable bit-rate, which gives an absolute performance measure, and is often of greater interest to protocol designers.

Here, we propose a unified normalized throughput metric, which allows the existing normalized throughput of both single and multi-hops to be expressed in a general formula. Since the current multi-hop metrics do not yield much insight on the best achievable bit-rate, we present a new variant, that benchmarks against the Maximum Achievable Throughput (MAT). The MAT-normalized throughput is characterized by the product of maximum number of successful simultaneous transmissions and link rate. To compute the former, a binary integer linear programming (BILP) problem is formulated. We next demonstrate the use of our metric in both string and grid topologies. We also derive exact mathematical expressions for the maximum successful simultaneous transmissions for these two topologies. Unlike existing metrics, our metric allows for better performance comparison across different MAC protocols.

II. Our Proposed Throughput Metric

A. The Unified Normalized Throughput Metric

We first summarize three existing throughput metrics that are commonly used for evaluating a given MAC protocol $\mathcal{P}$:

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- Aggregate throughput (in bps), $\gamma_{\text{ag}}(\mathcal{P})$:

$$\gamma_{\text{ag}}(\mathcal{P}) \triangleq \frac{\sum_{i=1}^{i=n} r_i(\mathcal{P}) \cdot L_{\text{DATA}}}{T}, \tag{1}$$

where n is the total number of nodes, L_{DATA} is the data packet's payload length in bits, $r_i(\mathcal{P})$ is the total number of data packets successfully received by destination i in a duration of T seconds; the total number of data packets received depends on the MAC protocol employed.

- Throughput per-node (in bps), $\gamma_{\text{nd}}(\mathcal{P})$:

$$\gamma_{\text{nd}}(\mathcal{P}) \triangleq \gamma_{\text{ag}}(\mathcal{P})/n. \tag{2}$$

- Rate-normalized throughput per-node (unitless), $\gamma_{\text{r}}(\mathcal{P})$:

$$\gamma_{\text{r}}(\mathcal{P}) \triangleq \gamma_{\text{nd}}(\mathcal{P})/R = \gamma_{\text{ag}}(\mathcal{P})/[n \cdot R]. \tag{3}$$

To complement the existing throughput metrics, we propose a unified normalized throughput metric, $\gamma_{\text{norm}}(\mathcal{P})$:

$$\gamma_{\text{norm}}(\mathcal{P}) \triangleq \gamma_{\text{ag}}(\mathcal{P})/[\beta \cdot R], \tag{4}$$

where $\beta > 0$ is a normalization factor. For single-hop networks, we set $\beta = 1$ and (4) reduces to the usual normalized throughput metric. For multi-hop networks, $\beta = 1/R$, $\beta = n/R$ and $\beta = n$ give (1), (2) and (3) respectively. Unlike the single-hop's throughput notion, these existing variants of the multi-hop throughput metric do not seek to normalize by the best achievable bit-rate. We therefore introduce the "MAT-normalized throughput", $\gamma_{\text{MAT}}(\mathcal{P})$, by setting $\beta = s_{\text{max}}$. Here, $0 \leq \gamma_{\text{MAT}}(\mathcal{P}) \leq 1$, and s_{max} is defined as the maximum number of successful simultaneous transmissions that can be supported by a given multi-hop network topology, for which all simultaneously transmitted data packets do not collide with each other. In other words, we normalize the computed aggregate throughput by maximum achievable throughput, which is the maximum aggregate data bit-rate characterized by the product of s_{max} and link rate R. Note that, $s_{\text{max}} \leq n/2$, since there could be at most $n/2$ number of simultaneous transmissions at any given time, due to the transceiver's half-duplex property. To further tighten the bound, we seek an *exact* maximum throughput by finding the optimal s_{max}, via an optimization approach in Section II-B. Compared to existing metrics, $\gamma_{\text{MAT}}(\mathcal{P})$ offers a clear quantitative indication of how close is the protocol's performance to what is best achievable, and is more useful in designing a better MAC protocol. Note, however, that we do not account for the protocol's fairness when computing s_{max}, which we intend to address in our future work. Nonetheless, it can be adopted for evaluating all classes of MAC protocols (e.g., contention-based MAC, schedule-based MAC, etc.), that do not enforce fairness.

B. The Binary Integer Linear Programming Formulation

Using a BILP optimization approach, we now explain how to compute s_{max} for a given network topology.

1) General Assumptions: We consider a static multi-hop wireless network with n homogenous nodes, and negligible propagation delay. Each node has a single omni-directional, half-duplex transceiver with link rate R. All nodes are arbitrarily placed and they communicate using a single-channel.

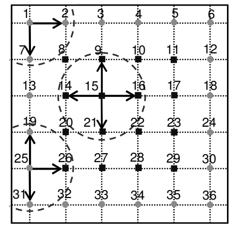


Fig. 1. A square grid (6×6) topology used in our evaluations. Every node is placed at the grid intersection point. Each of the non-boundary nodes (square node) has four one-hop neighbors. In contrast, each of the boundary nodes (round node) has fewer than four neighbors; for example, the neighbors of node 1 are nodes 2 and 7.

To obtain the maximum throughput, each node is assumed to be always backlogged and has packets destined to any of its one-hop neighbors. They also have a common and fixed communication range, which is the same as the interference range. Note that the formulation can also be modified accordingly if the interference range is assumed to be longer than the communication range. The channel is assumed to be error-free, and packet reception fails if and only if packets collide with each other. Although we ignore the effects of imperfect channel in this paper, it can be considered by multiplying (4) with a factor of $(1 - p_{\text{e}})$, where p_{e} is the packet error rate.

2) Problem Formulation: We denote the set of n nodes as $\mathcal{T} = \{1, \cdots, n\}$. A set of binary decision variables a_{ij} are introduced, in which $a_{ij} = 1$ if node i is scheduled to transmit to node j, and $a_{ij} = 0$ if otherwise. We define $\mathcal{N}(x)$ as the set of one-hop neighboring nodes of node x. We also define $a_x = \sum_{y \in \mathcal{N}(x)} a_{xy}$. Finally, we define M as a large number that is greater than $\sum_{k \in \mathcal{T}} a_k$. Our BILP formulation is presented as:

$$\text{maximize} \sum_{i \in \mathcal{T}} \sum_{j \in \mathcal{N}(i)} a_{ij} \tag{5}$$

subject to:

$$\sum_{j \in \mathcal{N}(i)} (a_{ij} + a_{ji}) \leq 1, \quad \forall i \in \mathcal{T} \tag{6}$$

$$a_{ij} + \sum_{k \in \mathcal{N}(j) \cup \{j\} \setminus \{i\}} (a_k) - (x_{ij} \cdot M) \leq 1, \quad \forall i \in \mathcal{T}, \forall j \in \mathcal{N}(i) \tag{7}$$

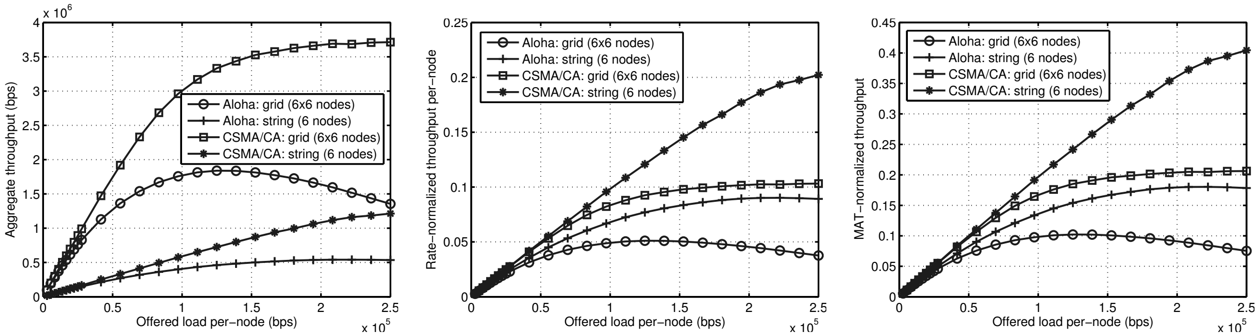
$$a_{ij} + x_{ij} = 1, \quad \forall i \in \mathcal{T}, \forall j \in \mathcal{N}(i) \tag{8}$$

$$a_{ij} = \{0, 1\};\ x_{ij} = \{0, 1\}, \quad \forall i \in \mathcal{T}, \forall j \in \mathcal{N}(i) \tag{9}$$

For the objective function in (5), we seek to maximize the total number of links (e.g., a_{ij}) that can be activated simultaneously. Since a node operates in half-duplex, constraint (6) ensures that it cannot transmit and receive at the same time. In addition, constraint (7) states that packet transmission from a sender node $i \in \mathcal{T}$ to its intended receiver node $j \in \mathcal{N}(i)$ is allowed if and only if the packet reception is free from interference at the receiver j (i.e., the receiver j and all its one-hop neighbors in $\mathcal{N}(j)$ must be inactive). Note that in (7), we also introduce a binary variable x_{ij} (as defined in (8)) to ensure that no unnecessary constraint is imposed on the sum of a_k by (7) if a_{ij} is inactive. The above BILP can be solved using a standard optimization solver such as CPLEX [11]. Finally, $s_{\text{max}} = \sum_{x \in \mathcal{T}} a_x$ from any optimal solution found.

C. Illustration Using Regular Structured Topologies

We illustrate the use of our metric using both string and square grid networks (see Fig. 1), which are commonly used



(a) (b) (c)

Fig. 2. Throughput comparisons of Aloha and CSMA/CA MAC protocols in both string (6 nodes) and square grid (6 × 6 nodes) topologies, by using different throughput metrics: (a) aggregate throughput, (b) rate-normalized throughput per-node, and (c) MAT-normalized throughput.

TABLE I
CPLEX'S SOLUTIONS OF s_{MAX} FOR STRING TOPOLOGY

n	1	2	3	4	5	6	7	8	9	10
$s_{\max}^{\text{string}}(n)$	-	1	1	2	2	3	3	4	4	5
n	11	12	13	14	15	16	17	18	19	20
$s_{\max}^{\text{string}}(n)$	5	6	6	7	7	8	8	9	9	10

TABLE II
CPLEX'S SOLUTIONS OF s_{MAX} FOR SQUARE GRID TOPOLOGY

d	1	2	3	4	5	6	7	8
$n = d \times d$	1	4	9	16	25	36	49	64
$s_{\max}^{\text{grid}}(n)$	-	2	4	8	11	18	22	32
d	9	10	11	12	13	14	15	16
$n = d \times d$	81	100	121	144	169	196	225	256
$s_{\max}^{\text{grid}}(n)$	37	50	56	72	79	98	106	128

for evaluating MAC protocols. Unless stated otherwise, we adopt the same assumptions as in Section II-B1, for both topologies. Note that, both topologies are *non-wraparound*.

1) Illustrating MAT-normalized throughput: Fig. 2(a)–2(c) show the use of different throughput metrics to evaluate Aloha and CSMA/CA (Carrier Sense Multiple Access with Collision Avoidance) protocols, in both string (6 nodes) and square grid (6×6 nodes) networks. Note that, from CPLEX solutions, $s_{\max}$ for these string and grid topologies are 3 and 18, respectively (denoted by $s_{\max}^{\text{string}}(6)$ and $s_{\max}^{\text{grid}}(36)$ in Table I and Table II, respectively). We also set $R = 1$ Mbps and $L_{\text{DATA}} = 1$ KB.

In Fig. 2(a), we observe that the aggregate throughput grows as the network size increases from 6 nodes to 36 nodes for both protocols, since a larger network could accommodate more simultaneous transmissions. However, the aggregate throughput metric does not illustrate the fact that a given MAC protocol experiences less contention in the string topology (i.e., due to less number of neighbors), compared to its counterpart in grid topology. This, however, is shown in both Fig. 2(b)–2(c), in which it is realized that a MAC actually performs more efficiently in the string case, compared to the grid topology.

More importantly, the use of our MAT-normalized throughput in Fig. 2(c) offers even more insights, as it can show how close is a protocol's performance from the maximum possible throughput for a given network topology. As an example, it can now be appreciated from Fig. 2(c) that Aloha's peak throughput for a multi-hop string topology is actually quite close to its theoretical peak for single-hop networks, which is around 18% of the maximum achievable throughput. In contrast, the metric in Fig. 2(b) may give a misleading perception that the Aloha's peak throughput is much lower (around 0.09). Note that, we have also simulated other string topologies with different sizes, and obtained similar conclusions.

2) $s_{\max}$ for both string and square grid topologies: Here, we derive the closed-form expressions of $s_{\max}$ for both string and square grid topologies. This will be useful for future MAC protocol designers to evaluate their protocols' performance using MAT-normalized throughput based on these two topologies, without the need to solve the BILP.

Theorem 1. $s_{\max}$ for a non-wraparound string topology of $\{n | n \in \mathbb{Z}^+$ and $n \neq 1\}$ nodes, $s_{\max}^{\text{string}}(n)$ is $\lfloor n/2 \rfloor$.

Proof: To ease our explanation, each node is given a unique ID (node IDs of 1 to n). We first show that the above expression is valid, when n is even. For $n = 2$, there is a single transmission. For $n = 4$, there can be 2 simultaneous transmissions; the transmission patterns are either $\{[1,2],[4,3]\}$ (as the 4-node case in Fig. 3), or $\{[2,1],[3,4]\}$, where $[x, y]$ denotes a sender x transmits to a receiver y. Note that, the sender-receiver (S-R) node pair of 3-4 has an "inverse" transmission pattern, with respect to that of their adjacent S-R pair of 1-2. Similarly, for $n = \{6, 8, 10, \ldots\}$, every subsequent two nodes further along the topology forms an S-R pair that assumes an inverse transmission pattern from the preceding pair, so as to allow maximal successful simultaneous transmissions (see Fig. 3). From these repeating patterns, it can be seen that for even n, $s_{\max}^{\text{string}}(n) = n/2 = \lfloor n/2 \rfloor$. Finally, when n is odd, the continuous string of $n-1$ nodes can form $(n-1)/2$ disjoint S-R pairs, thus leaving a residual node at the end of the topology (see Fig. 3). To yield the optimal number of transmissions without causing any collision, those S-R node pairs can also assume the inverse transmission pattern. Hence, for odd n, $s_{\max}^{\text{string}}(n) = (n-1)/2 = \lfloor n/2 \rfloor$. ■

Theorem 2. $s_{\max}$ for a non-wraparound square grid topology of $\{n | n = d \times d, d \in \mathbb{Z}^+$ and $d \neq 1\}$ nodes, $s_{\max}^{\text{grid}}(n)$ is:

$$s_{\max}^{\text{grid}}(n = d \times d) = \begin{cases} d^2/2, & \text{when } d \text{ is even,} \\ [d^2 - (d-2)]/2, & \text{when } d \text{ is odd.} \end{cases} \quad (10)$$

Proof: For a $d \times d$ square grid, we first show that $s_{\max}^{\text{grid}}(n = d \times d) = d^2/2$, when d is even. Now, let us assume that d number of nodes in the 1st-row of the grid topology employ the inverse transmission patterns, as is done in the string

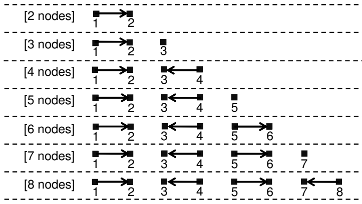


Fig. 3. Several cases of string topologies, and their respective possible simultaneous transmission patterns that yield the optimal number of transmissions.

topology case, so as to maximize the number of successful simultaneous transmissions. In this case, there are $d/2$ S-R node pairs that can be allowed to transmit simultaneously. Then, in the 2^{nd}-row of the grid, the set of nodes that are directly under the same columns as the receiving nodes in the 1^{st}-row, cannot transmit because this would interfere with the packet receptions of those receiving nodes in the 1^{st}-row (e.g., nodes 10, 11, 14, and 15 cannot transmit in our example in Fig. 4). Instead, they could receive from their respective adjacent neighbors; thus, the d number of nodes in the 2^{nd}-row also have the same inverse transmission patterns, as those nodes in the 1^{st}-row. Following this argument, it can be seen that the remaining rows of the square grid, will also have the same transmission patterns as the 1^{st}-row. Hence, we have $s^{\text{grid}}_{\text{max}}(n) = d/2 \times d$, when d is even.

Next, we show that $s^{\text{grid}}_{\text{max}}(n = d \times d) = [d^2 - (d-2)]/2$, when d is odd. For a 3×3 grid, the optimal number of transmissions is 4; these transmissions are characterized by a "square" pattern, in which 4 non-conflicting S-R node pairs encompass an idle node, as shown in Fig. 5(a). For cases of $d = \{5, 7, 9, \ldots\}$, it is found that a generic transmission pattern, which consists of both square patterns (of size 3×3) and inverse transmission patterns, always yields an optimal number of transmissions. As grid sizes grow, it can accommodate multiple adjacent square patterns, which share common S-R node pairs at their squares' boundary, so as to maximize the number of simultaneous transmissions (see Fig. 5(b); note that those square patterns can start from any one of the grid's corners). As illustrated, these square patterns divide the grid into two regions (i.e., lower and upper regions), where those nodes can assume the inverse transmission patterns, so as to avoid interfering with the squares' transmissions. Let us denote q as the number of square patterns supported in a $d \times d$ square grid. For $d = \{3, 5, 7, 9, \ldots\}$, q would be $\{1, 2, 3, 4, \ldots\}$, respectively; thus, we can express $q = (d-1)/2$. Based on the above generic optimal transmission pattern for odd d, $s^{\text{grid}}_{\text{max}}(n)$ can be computed as the summation of number of successful simultaneous transmissions in: (i) the square patterns (n_{sq}), (ii) the lower region (n_{lo}), and (iii) the upper region (n_{up}). Therefore, $s^{\text{grid}}_{\text{max}}(n)$ is computed as,

$$\begin{aligned} s^{\text{grid}}_{\text{max}}(n) &= n_{\text{sq}} + n_{\text{lo}} + n_{\text{up}} \\ &= (3q+1) + \sum_{i=1}^{q-1} i + \sum_{i=q-1}^{d-3} i \\ &= 4q + [(d-3)(d-2)]/2 = [d^2 - (d-2)]/2. \quad \blacksquare \end{aligned}$$

We note that the transmission pattern in Fig. 5(b) is just one of the many possible optimal solutions. This means that multi-hop routing can be achieved by alternating between different

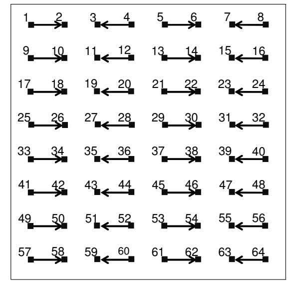
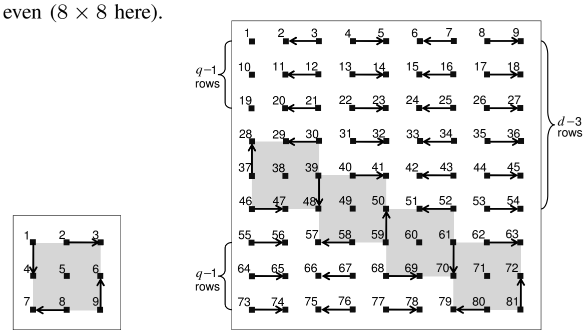


Fig. 4. A possible simultaneous transmission pattern that yields the optimal number of transmissions of $s^{\text{grid}}_{\text{max}}(64) = 32$ for the square grid topology when d is even (8×8 here).



(a) (b)

Fig. 5. The possible simultaneous transmission patterns that yield the optimal number of transmissions in the square grid topology when d is odd: (a) 3×3 square grid topology, where $s^{\text{grid}}_{\text{max}}(9) = 4$, (b) 9×9 square grid topology, where $s^{\text{grid}}_{\text{max}}(81) = 37$.

optimal transmission patterns.

III. Conclusion

To better evaluate MAC protocols in multi-hop networks, we proposed the MAT-normalized throughput metric. It offers more insights because it compares a protocol's throughput performance to the best achievable. This serves as a useful guideline for system designers to decide whether any potential protocol enhancement is worth the effort and allows for better performance comparison across different MAC protocols.

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