# Receiving Mutual Impedance between Two Parallel Dipole Antennas

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## Abstract

*A new mutual impedance, the receiving mutual impedance, between two parallel dipole antennas is introduced and defined rigorously. Its differences from the conventional mutual impedance are fully explained. The variation of the receiving mutual impedance with antenna separation is calculated. The superiority of the receiving mutual impedance over the conventional mutual impedance in array problems is demonstrated in a direction-finding problem.*

## 1. Introduction

In antenna array design, one of the critical problems is the mutual coupling effect between antenna elements. Traditionally, the mutual impedance was used to measure the mutual coupling effect. The traditional (or conventional) mutual impedance employs a definition similar to that used for the mutual impedance in circuit analysis. While it does give some convenience for the analysis of the mutual coupling problems [1], it is rather questionable as whether this circuit concept can give accurate results in a typical field problem such as antenna array analysis. In this paper, we present a study on the characterization of mutual coupling effect between two parallel dipole antennas. A new mutual impedance, called the receiving mutual impedance, is introduced and defined rigorously. Its superiority over the conventional mutual impedance in array problems is demonstrated in a direction-finding problem.

## 2. The receiving mutual impedance

The two dipole antennas to be studied are configured as a linear array with their axes along the z-axis as shown in Fig. 1. The two antennas are completely identical with length $L$ and wire radius $a$. They are separated along the x-axis by a distance $d$. An external source coming from a direction $\phi$ on the azimuth plane is to excite the antennas. Both dipoles are connected to a terminal load $Z_L$. The receiving mutual impedance, denoted by $Z_t^{12}$, between these two dipoles is defined as the ratio of the induced voltage $V_1$ across the terminal load of antenna 1 to the current $I_2$ on the terminal load of antenna 2 when the array is excited by the external source. That is,

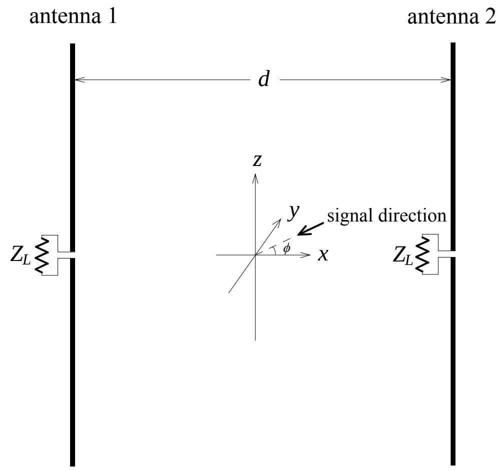

Fig. 1 The two dipole antennas and the external excitation signal source.

$$Z_t^{12} = \frac{V_1}{I_2}. \quad (1)$$

Here, $V_1$ on antenna 1 is that part of the induced voltage solely due to the excited current on antenna 2 and not due to the external source. This is an important condition placed on $V_1$, which makes the receiving mutual impedance different from the conventional mutual impedance. The word "receiving" is to distinguish this definition from the conventional definition of mutual impedance in that now both antennas are in the receiving mode. $Z_t^{21}$ is defined similarly but with the positions of antennas 1 and 2 interchanged. Note that this definition requires an external excitation source (as indicated in Fig. 1) to excite the two antennas. Furthermore, the definition of

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the receiving mutual impedance includes a terminal load $Z_L$ and hence it also depends on the terminal load connected to the antennas. The advantage of including the terminal load into the receiving mutual impedance is that the loading effect is also taken into account. This is similar to the definition of scattering parameters ($S$ parameters) which also depends on the system impedance.

To find the receiving mutual impedance, either experimental or theoretical methods can be used. The experimental method is given in [2] for the case of monopole antennas. In the theoretical method, the values of $V_1$ and $I_2$ in (1) can be calculated in an indirect way. First, the two dipoles are excited by the external source and the terminal voltages and currents on the two dipoles are calculated, for example denoted as $V'_1$, $V'_2$, $I_1$, and $I_2$. Secondly, the terminal voltages on two isolated dipoles are calculated. This can be done by removing one of the dipoles from the array when the terminal voltage on the other is being calculated. For example, the isolated terminal voltages are calculated to be $V''_1$ and $V''_2$. Then according to the superposition principle, the terminal voltage on antenna 1, i.e., $V_1$ in (1) is given by,

$$V_1 = V'_1 - V''_1 . \tag{2}$$

Hence $Z_t^{12}$ can be calculated by using (1). $Z_t^{21}$ can be calculated similarly.

The following points are the basic differences between the receiving and the conventional mutual impedances.

1. The receiving mutual impedance is defined with the two antennas in the receiving mode whereas in the conventional mutual impedance, one of the antennas must be in the transmitting mode. In typical application situations of receiving antenna arrays, the current distributions on the antenna elements when they are in the receiving mode are different from those when they are in the transmitting mode. However, the conventional mutual impedance ignores this difference and attempts to model the receiving and transmitting situations using a single model. When the antenna element sizes are small compared with the wavelength (within the quasi-static approximation used for circuit analysis), the calculation of the conventional mutual impedance is approximately correct because the current distributions on the antenna elements are relatively the same whether the antenna elements are in the receiving or transmitting mode. However, when the antenna sizes are comparable with the wavelength, for example half wavelength, the receiving and transmitting mode current distributions are quite different, especially the phases. In a word, the conventional mutual impedance fails to accurately model the real situation.
2. The receiving mutual impedance is defined with two loaded antennas whereas the conventional mutual impedance is defined with one antenna short circuited while the other open circuited. In the quasi-static approximation for circuit analysis, the form (shape) of the current distribution on a component (for example, a connecting wire) is unchanged when the circuit is connected to different loads. This gives a convenience to use the open-circuit voltage or short-circuit current in defining the mutual impedance because different loads only change the current level but not the form of the current distribution itself. However, from antenna analysis, we know that a different antenna load will give a different current distribution and which produces a different field distribution. This in turn results in a different mutual impedance. Hence, in a strict sense, the short-circuit open-circuit concept is not applicable to antenna analysis.
3. Considering from the network theory, the receiving mutual impedance is defined on a three-port system while the conventional mutual impedance is defined on a two-port system. The definition of the receiving mutual impedance requires an external source to excite the two antennas and hence one can consider the external source as the third port while the two receiving antennas as the first and the second ports. However, the conventional mutual impedance represents a closed two-port system and no external source is required. The consequence of this difference is that the conventional mutual impedance satisfies the reciprocity theorem, i.e., $Z_{12} = Z_{21}$, whereas the receiving mutual impedance does not, i.e., in general $Z_t^{12} \neq Z_t^{21}$.
4. The definition of the receiving mutual impedance includes an external source whereas the conventional mutual impedance does not. Because of this, the receiving mutual impedance makes a reference to the external signal environment, i.e., some information of the external signal source is built into the mutual impedance. This makes it more accurate to quantify the mutual coupling effect in a receiving signal environment. The conventional mutual impedance is defined in a closed system with no reference to the external signal environment. Hence it cannot accurately quantify the mutual coupling effect in a receiving situation.

The accurate determination of the receiving mutual impedance relies on the accurate determination of the current $I_2$ in (1). $I_2$ is the terminal current of the current distribution on antenna 2 excited by the signal source and the scattered field from antenna 1. Note that it is not just $I_2$ but the whole current distribution on antenna 2 that determines the coupled voltage $V_1$ on antenna 1. In order to obtain an accurate current distribution on antenna 2, it is important to have some information about the signal source, especially about its position or coming direction. It turns out that this is always possible for dipole antenna arrays as most of the applications involving the signal source/sources lying on a plane perpendicular (or nearly perpendicular) to the dipole axes. This is an important piece of information and can be used in the calculation of the receiving mutual impedance. It is further found that even the actually signal source may not exactly lie on this perpendicular plane, the error resulting from this mismatch is actually quite small [3], [4]. On the other hand, if the position of the signal source is exactly known, then the receiving mutual impedance is capable of almost exactly quantifying the mutual coupling effect, i.e., that exact amount of coupled voltage can be almost exactly known. This is the case in phased arrays used in magnetic resonance imaging (MRI), in which the signal source can be exactly known [5]. One may say that if none of the information regarding the signal source is available, how to calculate the receiving mutual impedance? Then under that situation, currently there are no accurate methods to determine the mutual coupling effect. However, one can always make an initial trial to gather some information regarding the signal source, for example a field test.

## 3. Results and discussions

Fig. 2 shows the receiving mutual impedance between two dipole antennas with different antenna separation. The dimensions of the dipole antennas are $d = 0.5\lambda, L = 0.5\lambda, a = \lambda/200$, and $Z_L = 50\Omega$. The results were obtained with the direction of the excitation source at an angle $\phi$ = 0° on the plane perpendicular to the dipole axes. The excitation source is a vertically polarized plane wave. The conventional mutual impedance $Z_{12}$ is also shown for comparison. It can be seen that the receiving mutual impedance and the conventional mutual impedance are quite different, especially when the antenna separation is small and the mutual coupling effect is large. It is interesting to compare the difference between the receiving and conventional mutual impedances as antenna separation $d$ approaches zero. For the conventional mutual impedance, by virtue of its definition, it approaches the self-impedance of the dipole as $d$ approaches zero. However, for the receiving mutual impedance, it does not approach the terminal load $Z_L$. The reason for this is that when $d$ approaches zero, the current distribution on antenna 2 (see (1)) will indeed approach the current distribution of a single antenna (same as when $d$ approaches infinity!). But by the definition of the receiving mutual impedance in (1), the coupled voltage $V_1$ is not equal to the total voltage induced by the current distribution on antenna 2 and the external excitation source. $V_1$ only accounts for the induced voltage caused by the current distribution on antenna 2 and not that caused by the external excitation source. Hence $Z_t^{12} \neq Z_L$ as $d$ approaches zero.

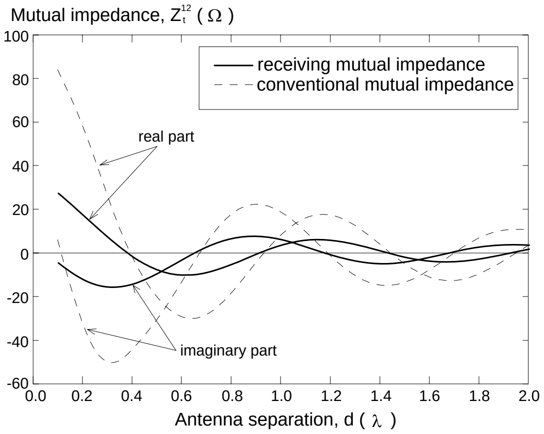

Fig. 2 The receiving mutual impedance between two parallel dipole antennas with antenna separation and with $d = 0.5\lambda$, $L = 0.5\lambda$, $a = \lambda/200$, and $Z_L = 50\Omega$.

As we mentioned earlier, the definition of the receiving mutual impedance requires an external excitation source (a plane electromagnetic wave in the above example), the receiving mutual impedance is in general dependent on the direction of the external source. However, for the case of dipole antennas, we found that the receiving mutual impedance is independent of the incident direction of the plane wave so far as it is lying on the plane perpendicular to the dipole axes. This is because the excitation wave only induces current distributions on the dipoles. The receiving mutual impedance actually accounts for the re-radiations from these current distributions (secondary sources) and not the radiation from the excitation wave (primary source). As dipole antennas have an omni-directional radiation pattern in the azimuth plane ($\phi$ direction), the induced current distribution on a dipole will be same for any incident angle of the excitation wave and hence the receiving mutual impedance is independent of $\phi$. This will be the same even when there is another dipole antenna nearby.

We have also obtained the results for $Z_t^{21}$. The symmetric relationship of the two-antenna configuration makes it no difference when the positions of the two antennas are interchanged. This means that the results of $Z_t^{21}$ are almost the same as $Z_t^{12}$ and hence they are not shown here.

## 4. Application to direction finding

It is well known that eigenstructure-based direction finding algorithms such as MUSIC are very sensitive to the effect of mutual coupling between the signal sensors [6]. In this section, the receiving mutual impedance defined above is used to quantify the mutual coupling effect in a dipole array which is used to detect the direction-of-arrivals (DOAs) of two coherent signals by using the MUSIC algorithm. The dipole array in this example consists of four identical dipole elements same as those in Fig. 2. The four dipoles are configured as a uniform linear array. The inter-element spacing is set to 0.5λ. The two coherent signals are plane electromagnetic waves with vertical polarization and come from the azimuth plane at $\phi$ = 30° and 57°. The receiving mutual impedances are calculated as described above. The coupled voltages in each antenna due to the other antennas in the array are then calculated using the receiving mutual impedances and removed from the terminal voltage (details can be found in [2]). The compensated voltages are then processed by MUSIC to extract the direction-of-arrival information of the two sources. An average signal-to-noise ratio of 10 dB is assumed. The detection result is shown in Fig. 3. Comparison is made with three other cases. The first case is the use of measured voltages which are the direct voltages obtained from antenna terminals. These voltages contain the coupled voltages. The second case is the use of the open-circuit voltages in which the coupled voltages have been removed using the conventional mutual impedance. The third case is the use of the ideal voltages which are the voltages obtained as when the antennas were completely isolated from each other, i.e., completely free of coupled voltages. It can be seen that using the compensated voltages, i.e., the voltages in which the coupled voltages have been removed using the receiving mutual impedance, the DOAs of the two signals can be accurately detected. However, using either the measured or the open-circuited voltages, it fails to detect the DOAs of the one or both of the signals.

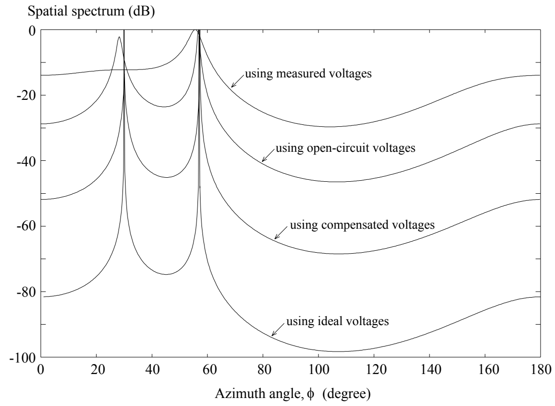

Fig. 3. The spatial spectrum of the MUSIC algorithm for the detection of the two coherent signals using different kinds of input voltages. The dimensions of the dipole array are: number of elements= 4, $d = 0.5\lambda$, $L = 0.5\lambda$, $a = \lambda/200$, and $Z_L = 50\Omega$.

## 5. Conclusions

A new definition of mutual impedance, the receiving mutual impedance, between two dipole antennas is introduced. The receiving mutual impedance differs significantly from the conventional mutual impedance in that it takes the operation mode of the antennas into consideration. It is shown that the receiving mutual impedance provides a more accurate description of the mutual coupling effect than the conventional mutual impedance. The formal definition of the receiving mutual impedance and the methods to obtain it have been given. The superiority of the receiving mutual impedance over the conventional mutual impedance in array problems has been fully demonstrated in a direction-finding problem.

## 6. References

[1] J. Gupta and A. A. Ksienski, "Effect of mutual coupling on the performance of adaptive arrays," *IEEE Trans. Antennas Propagat.*, vol. 31, pp. 785-791, Sept. 1983.

[2] H. T. Hui, ‘A new definition of mutual impedance for application in dipole receiving antenna arrays," *IEEE Antennas and Wireless Propagation Letters*, vol. 3, pp. 364-367, 2004.

[3] H. T. Hui, “A practical approach to compensate for the mutual coupling effect of an adaptive dipole array,” *IEEE Trans. Antennas Propag.*, vol. 52, pp. 1262-1269, 2004.

[4] H. T. Hui, “Improved compensation for the mutual coupling effect in a dipole array for direction finding,” *IEEE Trans. Antennas Propag.*, vol. 51, pp 2498-2503, 2003.

[5] H. T. Hui, ‘An effective compensation method for the mutual coupling effect in phased arrays for magnetic resonance imaging’, *IEEE Trans. Antennas Propag.*, vol. 53, pp. 3576-3583, 2005.

[6] B. Friedlander and A. J. Weiss, "Direction finding in the presence of mutual coupling," *IEEE Trans. Antennas Propagat.*, vol. 39, pp. 273-284, Mar. 1991.